

## REPRESENTATION OF ELASTIC DEFORMATIONS AS DEVIATION ZONE AND ITS CONNECTION WITH STATISTICAL TOLERANCE ZONES

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### 1. Introduction

Predictive engineering is playing a more and more important role in modern product development. The objective of which is to foresee the technical product behavior as accurately as possible and thereby, to make it controllable. Thus, already by means of a computer-internal model of the future product shall be tested how such product will behave under real conditions. Particularly in sectors commonly considered as difficult to handle and to see through, this trend continues. So for instance, commercial tolerance analysis products already exist, allowing (long) tolerance chains to be evaluated by statistical ways. As up to now, computations were only possible based on rigid body models while reality however is providing elastic material behavior, efforts are in progress to advance development in this direction. Industry [Glöggler 2004] as well as scientific institutions [Lustig 2005, Germer 2005, Von Praun 2002, Camelio 2003] intensively deal with this subject.

So a method was developed at the Chair of Engineering Design, which allows the interconnection of tolerances with elasticity information and the combined simulation of both [Lustig 2005, 2004, 2003]. Based on these cognitions and on existing internal dissertations [Hochmuth 2002], this paper presents another possibility of how to bring both parameters reasonably together.

### 2. Previous work and additional enhancement

Basis of the works introduced here is a software tool that originated at the Chair of Engineering Design. For the product developer, this tool provides the opportunity to have product-related stiffness calculations function-oriented evaluated [Hochmuth 2002]. In this case, evaluation takes place in a so-called "language of tolerances". Calculated displacements and deformed geometry elements for example are so transformed into cylindricity deviations. Taking this for a basis, this tool is advanced by the input of statistical tolerance information, in the following step enabling an interconnection between both parameters according to different strategies.

#### 2.1 Evaluation of product-related stiffness calculations

This type of evaluation analyzes the results of a finite element calculation in respect of tolerance zone description (ISO 1101) and a description of datum references (ISO 5459) for position tolerances. Following are substitution elements for functional areas generated as already known from metrology. On closer examination, thereof results an optimizing problem, as the number of nodal points needs to be broken down to the description of substituting elements. The type of such replacement elements can either be an enveloping element, a pen element, or an element developed in accordance with a minimum condition. In solving this optimizing problem, the Simplex method is applied. The exact sequence of this function-oriented evaluation is demonstrated in Figure 1.

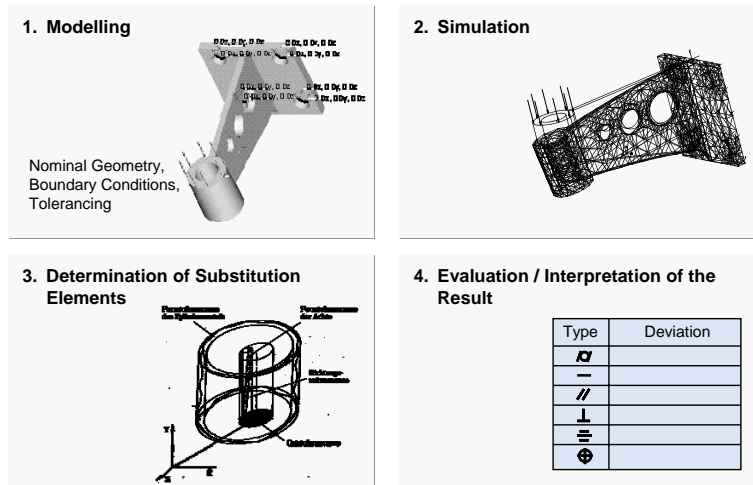


Figure 1. Concept of function-oriented evaluation

In a first step, model forming takes place, similar to a usual finite element analysis. Thereby the nominal geometry to be investigated must be on hand. Now the boundary conditions such as definition of clamping position, assignment of loading case, determination of the analysis type, and definition of the geometry elements to be analyzed can be applied. Then in the next step, the simulation of the entire system can take place, using an external equation solver. Thereafter, the substitution elements are determined, enabling the resulting displacements to be translated into the "language of tolerances".

### 2.2 Extensions to the hitherto approach

As already previously mentioned, it is necessary to commonly consider elasticities and tolerances in a computer-aided product analysis. However, this can only be realized if it becomes possible to describe both phenomena in one consistent language. The principal idea based on which this shall succeed is shown in Figure 2.

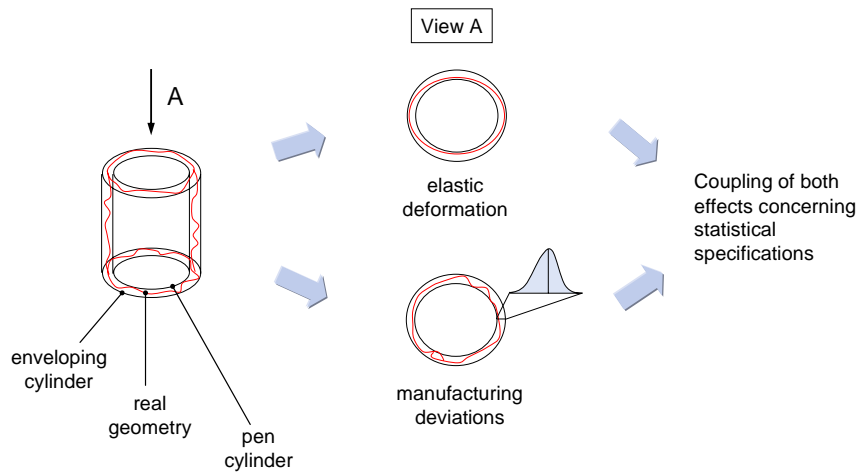


Figure 2. Basic idea of enhancement

At the left side, a deformed cylinder is shown together with its corresponding substitution elements, an enveloping and an inner pen cylinder. Viewing this scenario now from the top (View A) shows a picture that can be caused by both, elastic deformations as well as tolerances. The only difference is, for the elastically deformed geometry invariably the same condition applies i.e., simulation results

achieved under the same boundary conditions and loads are always identical. Different from tolerance examination, where the location of deviation-afflicted geometry always can only be indicated with a certain probability. Consequently, a common possibility to describe the situation exists and an interconnection of both phenomena can be realized.

So in addition to the procedure described under 2.1, also other modules need to be implemented. In addition to integrating more types of tolerances, the complete system must also provide the possibility to process deviations caused by manufacturing. Those are either experimentally determined or hypothetically assigned by the system according to various distribution types. Moreover, the program is to appropriately interconnect both phenomena, elastic deformation and deviations caused by manufacturing, and to compute an overall result. The output of an analysis report, in which all information essential for the user is comprehensibly prepared, rounds off the program.

### 3. Concept

Basis of this concept is a program developed at the Chair of Engineering Design with its functionality being demonstrated more detailed in [Hochmuth 2002]. Therefore in this chapter, only the newly added program parts shall be introduced more precisely.

As presented in [Hochmuth 2002], a solution for function-oriented evaluation of nodal point displacements already exists for some types of tolerances. Since implementation of this concept was realized under a meanwhile outdated version of Pro/ENGINEER™, existing solution must be transferred into the up-to-date. At the same time, the solution introduced in [Hochmuth 2002] is extended by the tolerance types straightness, circularity, and parallelism.

An inclusion of possible deviations caused by manufacturing can be achieved in two fundamentally different ways. Once it is possible to read out experimentally ascertained data being stored in the system and to provide them for superimposition. In many cases, such data exist in form of tables (Figure 3).

Größe Länge L der zu bearbeitenden Fläche		Ebenheitstoleranzen (µm)					Größe Länge L der zu bearbeitenden Fläche		Parallelitätstoleranzen (µm)				
Über (mm)	bis (mm)	beim Läppen	beim Schleifen	beim Fräsen	beim Drehen	beim Planieren	Über (mm)	bis (mm)	beim Drehen	beim Fräsen	beim Schleifen	bei Isolierpräzession	
	10	0,002	0,005	0,015	0,020	0,040		10	0,03	0,05	0,01	0,05	
10	25	0,004	0,015	0,030	0,040	0,080	10	25	0,05	0,05	0,02	0,10	
25	50	0,006	0,030	0,045	0,080	0,160	25	50	0,10	0,10	0,05	0,20	
50	120	0,010	0,050	0,080	0,120	0,280	50	120	0,10	0,15	0,08	0,20	
120	250	0,012	0,080	0,070	0,140	0,360	120	250	0,15	0,20	0,10	0,30	

Figure 3. Samples of experience values for shape and location tolerances [Source: Bosch]

Such experience values are statistical, determined by examining a great number of produced parts regarding their shape and location tolerances. Therefore, a statistical statement can be made, in as far such tolerances can be maintained in day-to-day production. In addition to coincidental occurrences, the determined values however depend very much on the respective production process. So for instance by lapping parts, closer tolerances can be achieved than by turning.

In cases where experimental data exist, the user may also input own numerical values in form of a mean value. As deviations caused by manufacturing rarely adopt the same value and always show a certain amount of variance, at that point an additional statistical distribution needs to be chosen according to which the input value varies. Consequently, different distribution functions such as normal, rectangular, or triangular distributions are realized here.

Since now both single values (elastic deviation and deviation caused by manufacturing) are on hand, an interconnection can be realized on a statistical basis.

For the purposes of a summarizing representation of results, an additional possibility is offered to have a report data file created, in which the boundary conditions for the simulation can be found as well as their results.

#### 4. Realization and implementation

The program for evaluating product-related stiffness calculations, acquisition of possible deviations caused by production, and superimposition of both effects was created using Pro/TOOLKIT™, the programming interface under Pro/ENGINEER™. Thereby, the new version Wildfire 2.0™ was accounted for. The complete analysis divides into four individual steps. Initial point of the whole is just modeled part geometry and a finite element simulation performed under MSC Nastran 2005™.

##### 4.1 Further processing of stiffness calculations

In a first step, function-oriented evaluation of the finite element nodal point displacement takes place. After the finite element analysis, geometry data are available in form of original and displaced nodal points, from which the resulting displacements can be determined. By means of model data from the CAD system, a clear allocation of nodal point coordinates to the respective shape element can be made.

The input data file for the FE software generated during preprocessing includes – among other – the nodal point coordinates of the undeformed networked part. After FE simulation using MSC Nastran 2005™, a data file with the nodal point coordinates of the distorted mesh exists as well as further information on the calculations made. In order not having to open both data files for each individual computation of the deviation calculation and to sort out essential data, to start with, the original node coordinates together with the respective displacements are written into a consolidated file. Furthermore, this procedure has the advantage that a simple integration of different output file formats into the read-in program become possible. In this way, data are always available in the same format for further program sequences.

Thereafter, the user can select a certain tolerance type and the formula elements required for it. By parameterization of geometry and representation in B-REP data structure, the nodes of ideal geometry and the node displacements are allocated to the formula elements considered. Then the function-oriented determination of load-conditional deviations takes place. In addition to the originally implemented procedures of "flatness", "position", and "cylindricity", another evaluation can be made in the system for "straightness", circularity", and "parallelism" (Table 1).

Table 1. Additional implementations

Type	Symbol	Name	Deviation zone	Form elements	Implemented
Form	t	Straightness	between 2 straight lines	real	<input checked="" type="checkbox"/>
			tubular	derived	<input type="checkbox"/>
Form	r	Flatness	between 2 planes	real	<input checked="" type="checkbox"/>
			between 2 planes	derived	<input type="checkbox"/>
Form	e	Circularity	between 2 circles	mainly real	<input checked="" type="checkbox"/>
Form	j	Cylindricity	between 2 cylinders	mainly real	<input checked="" type="checkbox"/>
Orientation	i	Parallelism	between 2 planes	real	<input checked="" type="checkbox"/>
			tubular	derived	<input type="checkbox"/>
Orientation	n	Perpendicularity	between 2 planes	real	<input type="checkbox"/>
			tubular	derived	<input type="checkbox"/>
Orientation	g	Angularity	between 2 planes	real	<input type="checkbox"/>
			tubular	derived	<input type="checkbox"/>
Location	l	Position	between 2 planes	real	<input type="checkbox"/>
			tubular	derived	<input checked="" type="checkbox"/>
Location	a	Concentricity	tubular	only axes	<input type="checkbox"/>
Location	d	Symmetry	between 2 planes	mainly derived	<input type="checkbox"/>
Runout	m	Circular Runout	ringshaped	real	<input type="checkbox"/>
Runout	h	Total Runout	ringshaped	real	<input type="checkbox"/>

#### 4.2 Integration of possible deviations caused by production

In a second step, it is possible to select potential deviations caused by production from a deposited database (experimentally determined) or to input self such data into the program (hypothetical assumptions) and to define a statistical distribution on which further calculations shall be based.

The experimentally determined deviations caused by production provided by Bosch (Figure 3) were implemented in the system as an example. As soon as this menu item is selected, the user is required to input additional, continuative information regarding the type of deviation caused by manufacturing. Accordingly, some more input information regarding the type of tolerance, magnitude of the considered geometry element, and the intended production method is queried from the user.

In case no experimental data exist, it is also possible to integrate hypothetical data into the system. Such data can be characterized on one hand by a numeric value that indicates permissible deviations and on the other by selecting a statistical distribution, which determines the variance. Implemented distributions are worst-case, normal, triangular, and rectangular distributions.

#### 4.3 Interconnection of both phenomena and representation of results

When both influences are determined, an interconnection of both parameters can be realized (step 3). It is topmost unlikely for the "worst case" to practically occur. As a general rule, the real result of the interconnection of both parameters is less. The reason for that is, all dimensions of produced lots – and consequently also their deviations – show a statistical distribution. Depending on the manufacturing process, one of the distribution types normal, triangular, or rectangular is allocated to each individual deviation. With this knowledge and a specified permissible lot fraction defective, the interconnection of both parameters is realized so that a statement regarding the total deviation can be made with a certain probability. Depending on the respective form of distribution of both influencing variables forming the basis, different approaches are applied in calculation. By means of convolution and the products resulting thereof as well as the fundamentals of statistical tolerating, the resulting statistical total variance can be calculated from both individual deviations.

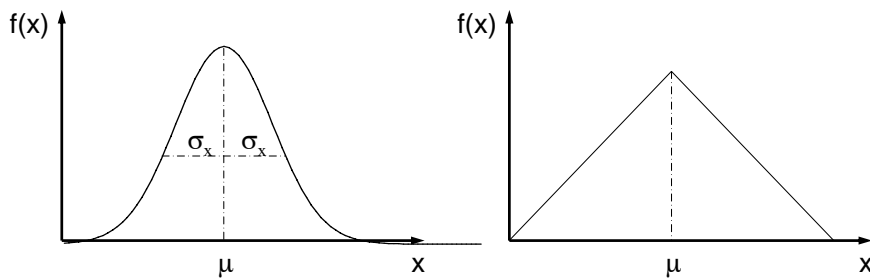


Figure 4. Standard and triangular distribution

Exists for example rectangular distribution for individual deviations, then the statistical total variance is calculated as follows:

$$t_s = \sum_{i=1}^n t_i - 2 \cdot \sqrt{2 \cdot F(z) \cdot \prod_{i=1}^n t_i} \quad (1)$$

with:  $t_i$  individual deviations

$F(z)$  one-sided error area under the distribution.

For triangular distribution (figure 4), the total variance results as

$$t_s = 6 \cdot \sigma_0 \quad (2)$$

whereas

$$\sigma_0 = \sqrt{\sum_{i=1}^n \sigma_i^2} \quad (3)$$

and

$$\sigma_i = \frac{t_i}{2\sqrt{6}} \quad (4)$$

with:  $\sigma_0$  variance of approximated standard distribution  
 $\sigma_i$  variance of individual triangular distributions  
 $t_i$  individual deviations.

For the case of standard distributions (figure 4) forming basis for the respective deviations, the total variance follows as:

$$t_s = 6 \cdot \sigma_0 \quad (5)$$

whereas

$$\sigma_0 = \sqrt{\sum_{i=1}^n \sigma_i^2} \quad (6)$$

and

$$\sigma_i = \frac{t_i}{6} \quad (7)$$

As soon as the interconnection is successfully implemented, the report function (step 4) is available to the user, which can be used to generate a report data file containing the most important calculation data. In addition to the three calculation results (deformation of the geometry element selected in "language of tolerances", introduced deviations caused by manufacturing, and result of the interconnected influence with probability statement) also the boundary conditions of the analysis (evaluation procedure and type of probability distribution) are listed again in detail.

The implemented program sequence is demonstrated in Figure 5.

## 5. Conclusion

The interest in superimposing elasticities and tolerances is rather great. Various concepts, so also one developed at the Chair of Engineering Design, try to combine both factors under one uniform describing language. The implementation of such cognitions in computer-aided tools however is presently not yet solved satisfactorily. A few software producers offer programs for the combined analysis of tolerances and elastic deformations with functionalities, which by far don't come close yet to the requirements of product development. Possible application areas exist for example in the automobile industry where the assembly of sheet metal parts is conditioned by elastic deformations.

With this existing, prototypically realized add-on program for Pro/ENGINEER™, a possibility shall be presented for computer-aided evaluation of tolerance zones and elasticities, in which important cognitions of previous [Hochmuth 2002] as well as current research work [Lustig 2005] also incorporate.

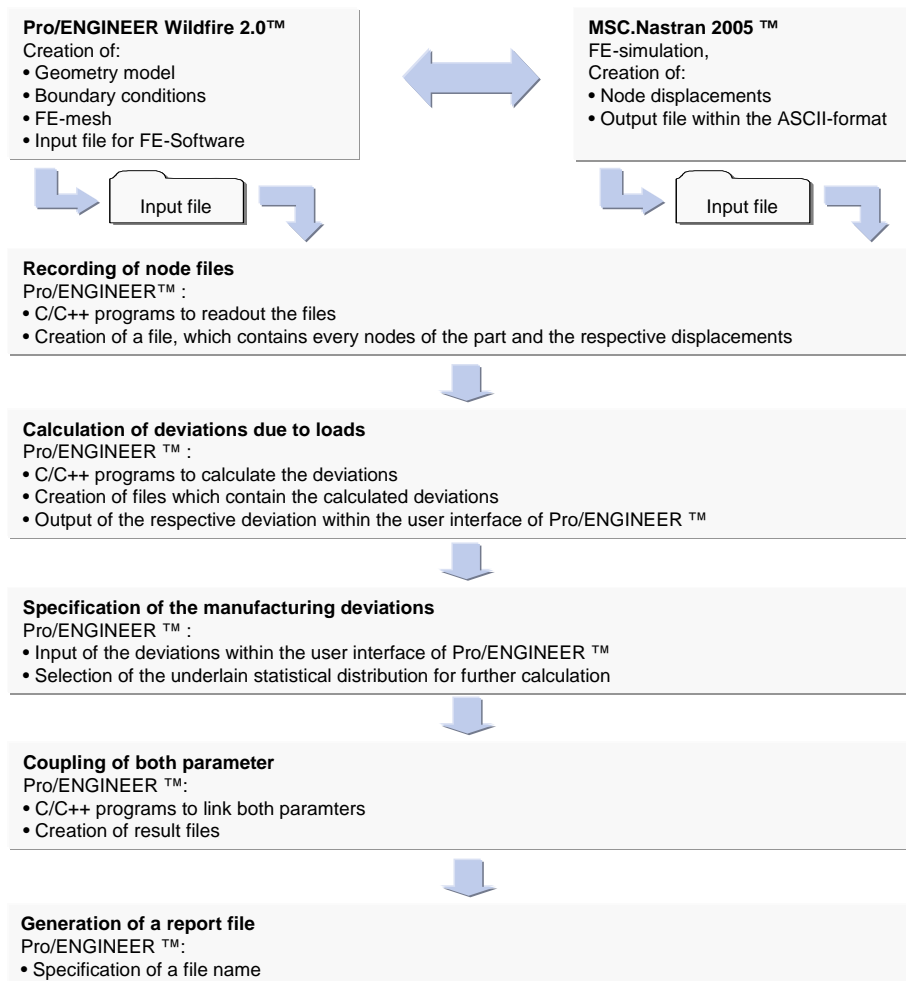


Figure 5. Extended program sequence

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