DESIGN 2006

DEVELOPING OF THE BEARING ORGANS COMPUTATIONAL MODULE FOR GEAR BOXES OPTIMALISATION SYSTEM

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Keywords: design engineering, technical systems, machine elements, properties, dynamics, gearing systems, bearing organs

1. Introduction

By the solution of the problems of the mathematical modelling of gear boxes predetermined particularly for regional railway vehicles, i.e. the dynamical behaviour of shaft type assembly systems (STA) the models of particular organs contained in shaft assembly groups (STG) have to be crated. The model of the gear organ (BO) is one of the most significant representatives of these organs and its correctness and accuracy participate significantly in the resulting behaviour of the whole system. The BO model with radial clearances is used in the presented computational system. The submodulus for the computation of are contained in this model. To gain the correct stiffness characteristics information at the gear producers is problematic matter because this information is commonly the trade secret. That is the main reason for the development of the computational submodulus enabling to compute the stiffness properties parameters of BO respecting the concrete loading cases of the whole STA system.

2. Mathematical model of shaft type assembly system

STA was decomposed into STG. STG is considered as a set of machine elements, which form logical units at the creation of the mathematical models of shaft systems along with gear wheels. STG were modelled by FME as 1-D continua [Slavik 1997] and were coupled by linear and time invariable tooth couplings. The model STA, created hereby, was bounded to the frame by bearing couplings with radial clearances. Every STA is composed of the machine elements and its organ structure is shown in Fig. 1.

2.1 Addressing of individual STG within the STA

In order to form a system, enabling to model the individual STG by the mathematical way, the address code for unique identification of every STG had to be find (Fig. 2):
STG a_c
a = axis number of the GS; c = consecutive number of the STG on axis "a"
Considering to the restricted area and focus of this contribution, we will solve the modelling of supporting and bearing organs only.

2.2 Modelling of the supporting shaft organs (SSO)

Every SSO is assembled by certain number of finite elements of the shaft type (FEST) bounded by nodal points (NP) (Fig. 3).
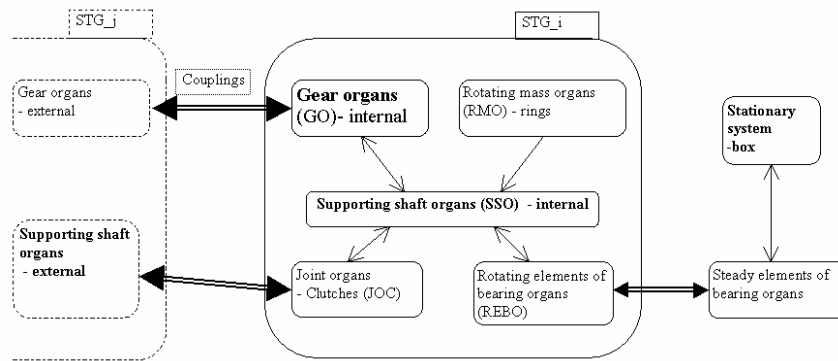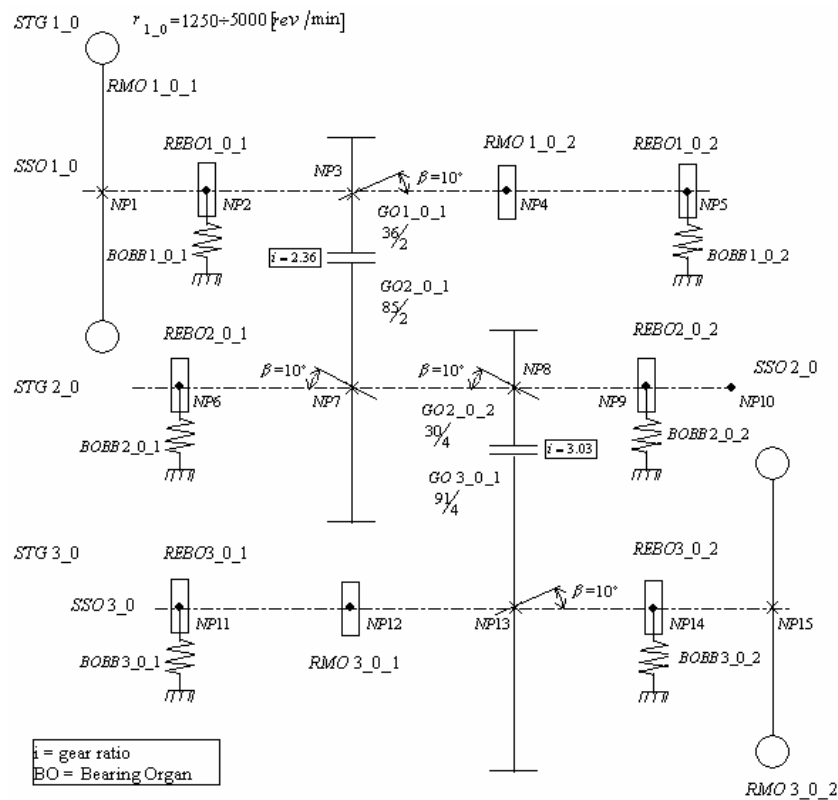
Figure 1. Organ structure of STA

Figure 2. Organ structure of gear system in a form of kinematic pattern

Addressing of SSO Elements:

FESTa_c_1…j   1…j   ordinal number of FEST in view of every STG
NP a_c_x…n   x…n   ordinal number of NP in view of the whole shaft system

2.2.1 Indication of bindings in nodal points

In Fig. 4, the connection of the organs O1 and O2 using unidirectional link is demonstrated. Unidirectional connection indicates, that the organ O1 acts on the organ O2 and through this organ affects the other organs.
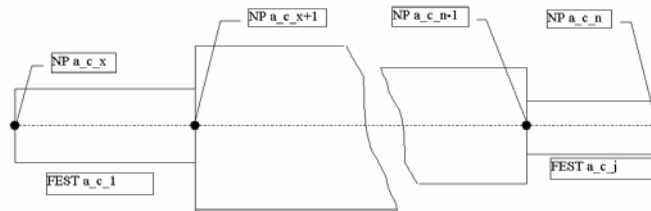
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Figure 3. Structure SSO

However, the organ O2 does not affect other organs through the organ O1. For example, such type of link is the link of rotating mass organ to SSO in NP (Fig.6).

In Fig. 5 bi-directional link is demonstrated. Linked organs affect other organs in both directions. Such type of link is for example the link of gear organ (GO) to SSO in NP (Fig.7).
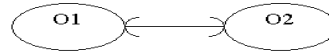

Figure 4. Unidirectional link

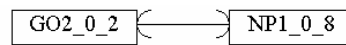Figure 5. Bi-directional link

Figure 6. Link RMO k SSO v NP

Figure 7. LinkGO k SSO v NP

As one NP can be common for two FESTs, it is necessary to accept consecutive agreement for link definition:

In view of link in a nodal point, common for two FESTa_c_j and FESTa_c_j+1, these links is assigned just to the nodal point belonging to FESTa_c_j.

2.3 Computational module for the mathematical modelling of the bearing organs (BO)

Two models have been created for mathematical description of BO, namely the model BO without consideration of the radial clearance (the common form by modelling of BO) and the model BO with consideration of the radial clearance.

Addressing of BO:

The bearing organs are bound to individual SSO in the appropriate NP (Fig.8).

Figure 8. Link BO to SSO in NP

BOXXi/an a_c_1…j]/[

bearing XX type indication

BB   - single-row ball bearing
OB   - single-row ball bearing with oblique-angled contact
RB   - roller bearing
DSB   - double-row spherical-roller bearing
i/an   - i = izotropic BO/ a = anizotropic BO

Addressing of BO

a/c   - see above
1…j   - ordinal number of BO in framework of STG

Direction, in which an axial force Bo is carried:

$]/[$ - BO carry the force Fa in a given direction $\vec{F}_a] / [\vec{F}_a$

Simplified assumption

- The rotating part of BO is modelled as a component of the rotating SSO (REBO).
- The unrotating part of BO is modelled as a component of bounded unrotating parts (e.g. box of gearing system)

2.3.1 The modelling of the elastic properties BO with respect the radial clearance $V_r = \Delta$

For every BO the S_i node must lie on the SSO axis and the centre of BO (rolling bearing).
There is assumption, that the radial and axial contact of the rolling bearing elements is fixed in the points H_j laying on the radius r_i, and their radius vector form an angle δ_{ij} wit the axis y (Fig.9).
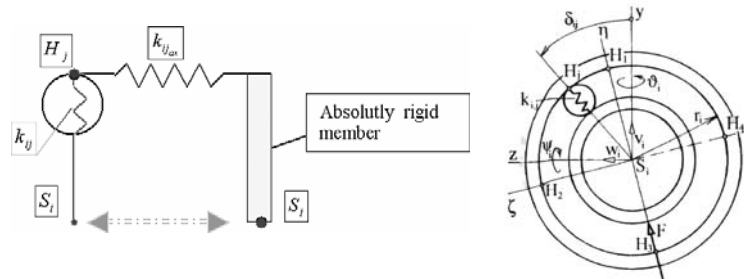

Figure 9. Function structure BO [3]

The general stiffness matrix of all bearing connections between shaft and body is:

$$\mathbf{K}^{LV}_c = \sum_i \sum_j k_{i,j} \begin{bmatrix} & \vdots & & \vdots & \\ \cdots & \mathbf{C}^{(i,j)}_{SS} & \cdots & \mathbf{C}^{(i,j)}_{SH} & \cdots \\ & \vdots & & \vdots & \\ \cdots & \mathbf{C}^{(i,j)}_{HS} & \cdots & \mathbf{C}^{(i,j)}_{HH} & \cdots \\ & \vdots & & \vdots & \end{bmatrix} \quad (1)$$

The assumption is possible that during the bearing rotation the rolling elements runs along the whole trajectory on the outer ring and forms the imaginary continuous ring. On this assumption we can compute the stiffness in any direction $\overrightarrow{S_i H_j}$ by the continual weight function (2) (Fig.10).

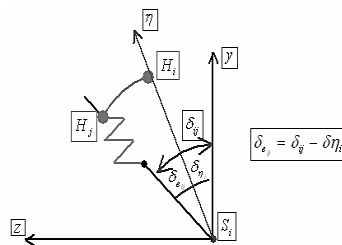**Figure 10. Angle $\delta_{e_{ij}}$**

$$\phi_i = 1 - tg\left(\frac{\delta_{ei,j}}{4}\right) \rightarrow \text{to round to one decimal point} \quad (2)$$

Then the stiffness of the bearing i v point j:

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$$k_{i,j} = \phi_i \cdot k_\eta \tag{3}$$

In this case we simplify the problem:
Isotropous BO is assumed. We centralize the axial contact between the rolling elements and the body into four points distributed evenly along the perimeter. Assuming the radial clearance Δ in the bearing, then:

$$k_{i,1} = k_\eta, \ k_{i,2} = k_{i,4} = 0,5\, k_\eta, \ k_{i,3} = 0, \ k^{ax}_{i,j} = 0,25\, k_{ax} \tag{4}$$

k_η - radial stiffness of the bearing
k_{ax} - axial stiffness of the bearing

2.3.2. The elastic movement of the bearing rings by the combined loading:

α = contact angle of the bearing; ψ = angle for the computation of the ring movement; D = ring diameter, δr = relative ring movement in the radial direction; δa = relative ring movement in the axial direction; $\delta\psi$ = general elastic movement of the rolling element in direction of angle ψ; ε = loading parameter

The general elastic movement in direction of angle ψ (Fig.11)

$$\delta\psi = \delta 1 + \delta 2 \tag{5}$$

$$\cos\psi 1 = -\frac{\delta_a}{\delta_r} \cdot tg\,\alpha \tag{6}$$

In the plain of symmetry for $\psi = 0$ the greatest depression is

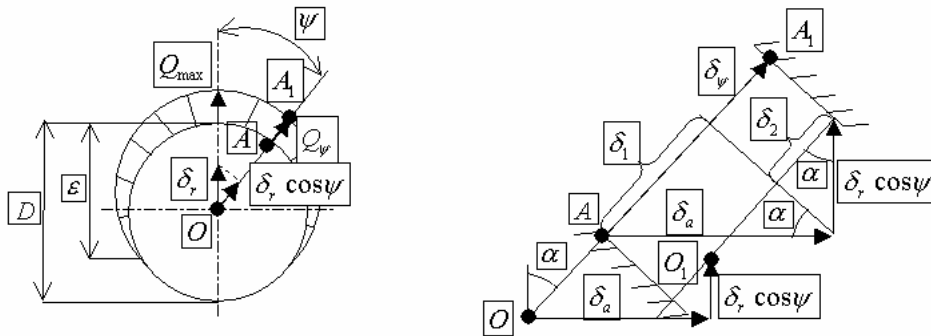$$\delta max = \delta r \cos\alpha + \delta a \sin\alpha \tag{7}$$

Figure 11. The elastic movement of the rolling element

From equation (5) a (7) is

$$\frac{\delta_\psi}{\delta_{max}} = 1 - \frac{(1-\cos\psi)}{1+\frac{\delta_a}{\delta_r}tg\alpha} \tag{8}$$

by designation

$$2\varepsilon = 1 + \frac{\delta_a}{\delta_r}\ \text{tg}\ \alpha \tag{9}$$

then

$$\frac{\delta_\psi}{\delta_{max}} = 1 - \frac{1}{2\varepsilon}\ (1 - \cos\psi) \tag{10}$$

if $\varepsilon \leq 1$ then the perimeter is loaded partially; if $\varepsilon = 0,5$ then the ring movement is radial only; if $\varepsilon = \infty$ then the ring movement is axial only. The movement in radial and axial direction:

$$\delta r = \frac{1}{2\varepsilon}\frac{\delta_{max}}{\cos\alpha}\ ;\quad \delta a = \frac{(2\varepsilon - 1)\delta_{max}}{2\varepsilon}\frac{}{\sin\alpha} \tag{11}$$

The bearing with radial clearance, radially loaded:

$$\frac{\delta_\psi}{\delta_{max}} = (1 - \frac{1}{2\varepsilon}(1 - \cos\psi)) \tag{12}$$

$$\varepsilon = \frac{1}{2}(1 - \frac{V_r}{2\delta_{max} + V_r}) = \frac{\delta_{max}}{2\delta_{max} + V_r} \tag{13}$$
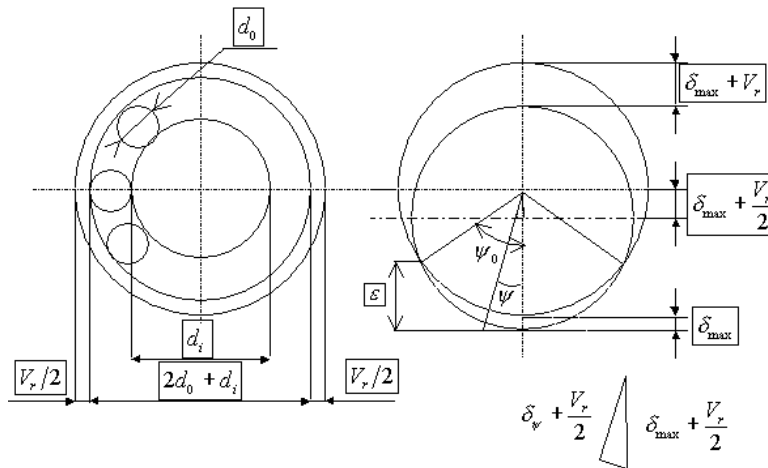

Figure 12. BO with radial clearance Vr – the function structure

The computation of the maximal loading of the rolling element:
Along the Hertz formulas the loading of the any body is given by the elastic deformations of the contact locations:

$$\frac{Q_\psi}{Q_{max}} = \left(\frac{\delta_\psi}{\delta_{max}}\right)^t\quad t = 3/2\quad\text{for ball bearings};\ t = 10/9\ \text{for roller bearings} \tag{14}$$

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The sum of forces loading the rolling elements in the radial and axial direction forms the radial and axial component of the load:

z = number of rolling elements in the bearing

$$Fr = \Sigma Q\psi \cos\psi \cos\alpha = Jr . z . Qmax . \cos\alpha \tag{15}$$

$$Fa = \Sigma Q\psi \sin\alpha = Ja . z . Qmax . \sin\alpha \tag{16}$$

Jr, Ja - radial (axial) integral – well approximated by Siovall:

$$Jr(\varepsilon) = \frac{1}{2\pi}\int_{-\psi_1}^{\psi_1}\frac{Q_\psi}{Q_{max}}\cos\psi\, d\psi \;,\; \psi1 = \arccos(1-2\varepsilon),\, \varepsilon \leq 1;\, \psi1 = \pi,\, \varepsilon > 1 \tag{17}$$

These integrals are depended on the load parameter ε only and they have the meaning of the dimensionless coefficients.

They serve for the computation of the most loaded rolling element Qmax:

$$Qmax = \frac{F_r}{J_r . z\cos\alpha}, nebo Q_{max} = \frac{F_a}{J_a . z\sin\alpha} \tag{18}$$

The general elastic movement is the sum of the deformations in the contact points between the rolling element and internal and external ring.

$$\delta c = \delta i + \delta e, \delta_{max} = \delta i_{max} + \delta e_{max} \tag{19}$$

$$\delta_{i,e} = 2,77 . 10^{-4} . \left(\frac{2\kappa}{\pi m_a}\right)_{i,e} \left(\Sigma\rho_{i,e}.Q^2\right)^{1/3} ; \text{ contact steel/ steel} \tag{20}$$

Coefficient $\left(\dfrac{2\kappa}{\pi m_a}\right)$ can be computed by the Hertz theory using the value $F(\rho)$ [Fröhlich 1980]

- κ is the general elliptic integral of the first order

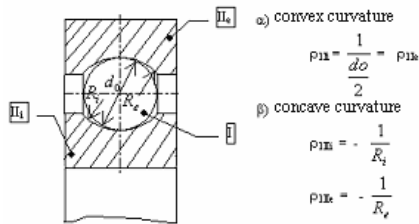The sum of the main twists of both bodies: (ball x ring) – here only for ball bearings

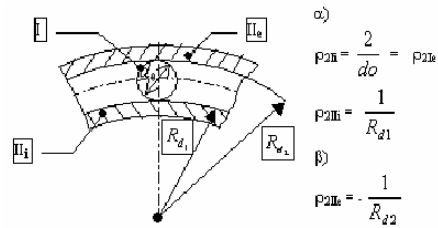

Figure 13. The section in BO by main plain 1   Figure 14. The sec. in BO by main plain 2

- internal ring   - external ring

$$\Sigma\rho i = \rho1Ii + \rho2Ii + \rho1IIi + \rho2Iii \qquad \Sigma\rho e = \rho1Ie + \rho2Ie + \rho1IIe + \rho2Iie \tag{21}$$

2.3.3 The contact stiffness LO in radial and axial direction:

$$k_\eta = F_r / \delta_{r_{max}} \qquad k_{ax} = F_a / \delta_{a_{max}} \quad (\text{see. } (4)) \tag{22}$$

3. Conclusions

The presented practice enables to compute the contact stiffness of BO in radial and axial direction and to form the mathematical model of BO with radial clearance. The modules in MATLAB have been included in the compact system (CS) enabling to evaluate the dynamic properties of the gear systems (GS) and to optimise them along the given requirements. In our opinion, the crucial contribution of the presented method is, that the analyses of gear systems are not maintained on the level of mathematical models only, but on level of the engineering design models. The CS was successfully used throughout multicriteria optimisation process of the GS, produced by the Skoda Transportation Company and mounted to the electrical drive unit 471 and locomotive 109E, which are predetermined for Czech railways.

Acknowledgement

References

Vanek,V.,Nemec, L., Hosnedl, S.: Design Science For Mathematical Modelling of Gear Systems. Dubrovník: Design 02, 2002

Fröhlich,J.: Technika uložení s valivými ložisky. Praha: SNTL, 1980

Slavik, J., Stejskal,V., Zeman, V.: Zaklady dynamiky stroju. (The Bases of Machine Dynamics). Prague (CZ): CVUT, 1997.

Xinwen N., Limin Z., Han D.: New statistical moments for the detection of defects in rolling element bearings. International Journal of Advanced Manufacturing Technology , vol.26, no.11-12 , Page: 1268-74, Springer-Verlag , 2005

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