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NUMERICAL ANALYSIS OF NON LINEAR THERMOSENSIBLE VISCOELASTIC SYSTEMS

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1. Introduction

This work can be placed within a wide research project undertaken by the Mechanical Engineering Department of the University of Brescia, which has the purpose to characterize the dynamic behaviour of rubber-to-metal devices used in the automotive industry. These objects, which contain viscoelstic materials such as elastomers, work in extreme conditions by the point of view of vibrations and thermical shocks. Moreover their behaviour can hardly be defined in an analytical way, because of their strong non linearity properties, thermosensibility and viscoelasticity (which causes internal heating as a consequence of energetic dissipation during vibrational phenomena). On the other hand, because of the continuous technical progress in the automotive field, today the need of preventing the behaviour of this components with a high level of precision is becoming more and more a crucial point. Neverthenless the purpose cannot be achieved using conventional methods. Infact: an analytical approach can't be used, because there aren't constitutive equations able to represent the material behaviour in all the working conditions; FEM softwares cannot give satisfactory results, because they don't consider the changes of material behaviour due to internal heating.
So the objective of the present project is delineating a non conventional way to solve this problem, realizing a software to simulate the behaviour of these systems.
This goal has been achieved through three different stages.

1. The characterization of dynamic and thermal properties of used materials (elastomers).
2. The searching of theoretical models, which are able to characterize the physic behaviour of used materials (elastomers).
3. The realizing of an automatic method of numerical calculus.

The first stage allows to characterize the behaviour of simple geometry test pieces, pointing out few parameters about the material viscoelasticity. In this stage also complex systems are evalued (deriving of dynamic amplification curves). The material parameters can be used to set with correct values the theoretical models, that the second stage tries to find out. In the third stage the adjusted models are involved by an automatic calculus code (EVA) to simulate numerically the behaviour of complex systems; the results of this process can be compared with those taken from the experimentation.
This method can be thought as a valid mean to be used for design development and optimisation.

2. Viscoelastic material thermo-mechanical characterization.

Mechanical Engineering Department of Brescia University has developed an innovative matherial and devices characterisation method based on the use of an electrodynamic shaker. This method allows to test elastomeric material in the frequencies range of 20÷400Hz, with dynamic deformation amplitudes

of 0.2%, 0.68, 0.5%, 0.7%, constant temperatures throughout each test but variable from test to test in the range 20°C ÷50°C and static pre-strain in the range of 0-10%.
This method has also been successfully compared with traditional method (DMTA - Dynamic Mechanical Thermal Analyzer) as shown in other papers (see references).

2.1 Format

In figure 1 the testing device is shown. Four rubber cylinder are mounted between a steel block and the plate of an electrodynamic shaker. By the analisys of the dynamic response at the shaker solicitation it is possible to measure the dynamic stiffness of the rubber. Changing temperature, deformation level and giving a static pre-strain to the rubber it is possible testing and so characterizing material dynamic behaviour Vs. these quantity.

Figure 1. Testing device

Figure 2 shows some experimental results obtained using the approach described.

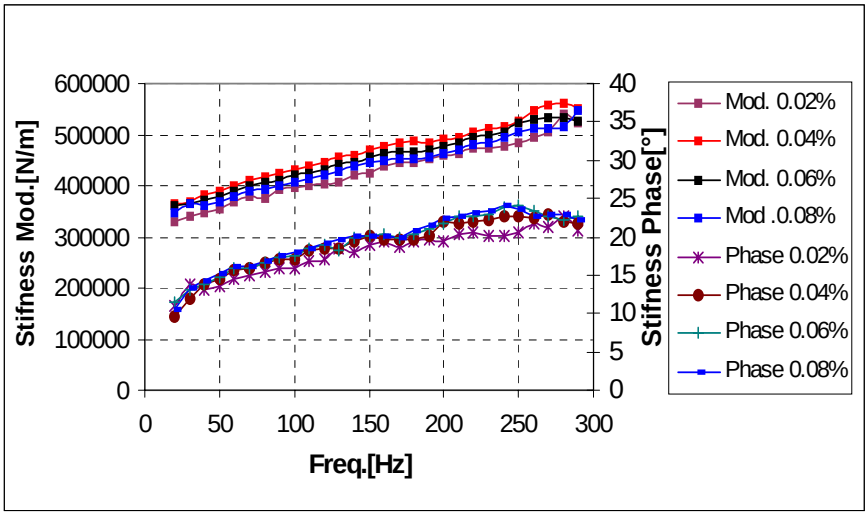

Figure 2. Material dynamic stiffness Vs. amplitude

3. Theoretical models

The elastomeric part of the vibrating system can be outlined using a concentrated parameters model to approximate its mechanical and thermal behaviour.
In this kind of models mass has to be considered as accumulated in a certain number of points (Figure 3a), while elasticity and viscosity are, not distributed characteristics of the continuous, but properties

assembled in some elements (springs and dampers) equipped with defined geometric properties, dependent from the particular system.

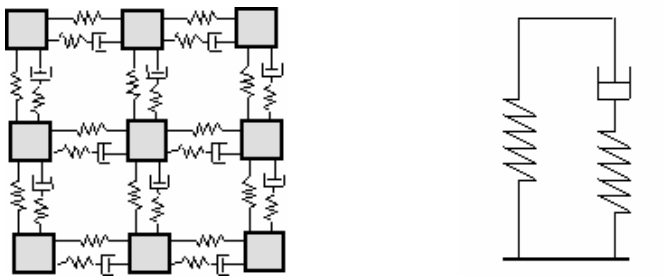

Figure 3. a) Discrete rubber model - b) Kelvin-Voigt model

In scientific literature about material science it is possible to find some different kinds of concentrated parameters models, describing elastomers. In this study, the Kelvin-Voigt model (Figure 3b) has been taken as reference; traditionally this model is used to simulate the behaviour of a whole component made of elastomer and allows to consider the typical frequency non linearity of these materials. The alternative point of view proposed in this work consist in considering that a generic component can be divided into elementary volume cells, linked by local Kelvin-Voigt elements. Following this approach, the single Kelvin-Voigt link doesn't describe the behaviour of the whole component, but only of a single "finite" element. So the global viscoelastic characteristic curve of a whole component is given by the union of the single local ones (Figure 3a). It is to be intended that every link applies an action along its own direction (axial) and also an action along the perpendicular to it (transversal).
In this study two possible models are taken into consideration. The first is based, as shown, on the Kelvin-Voigt configuration, while the second is based on the viscoelastic model, given by the parallel coupling of a spring and a dumper. At this time our research team is making some efforts to point out how to set an adjusted curve for the viscous dumper, trying to consider in a correct way the frequency non linearity.
The thermal behaviour is evaluated considering that the local mass elements can interact by concentrated thermal links, which are characterized by values depending from the particular material taken into consideration; the internal heating, typical of elastomers, can be evaluated considering power dissipated by the dampers.
The described method isn't linked to the use of one particular model, so it can be considered general.

4. Automatic calculus

The study of vibrating systems takes to the individuation of some differential equations describing the movement in the time dominion. These equations are solved using different methods of numeric integration (one-step, multi-step, implicit, explicit), which are implemented in the time integration cycle; so it's possible to draw the time history of every mass element in the analysed system, starting from the initial conditions.
The vibrating systems taken into consideration are typically made of rubber elements connected with steel components, which are considered as single bodies . So in the generic system there are different mass pieces:

- mass bits: they represent the division of the elastomeric mass into elementary cells; the mass is considered to be concentrated in points, so these parts have only three degrees of freedom in the space (translations);
- single bodies: they represent bodies which are much more stiff than rubber (steel); they have six degrees of freedom (three translations and three rotations);
- bonds: they have six possible movements, but these are given.

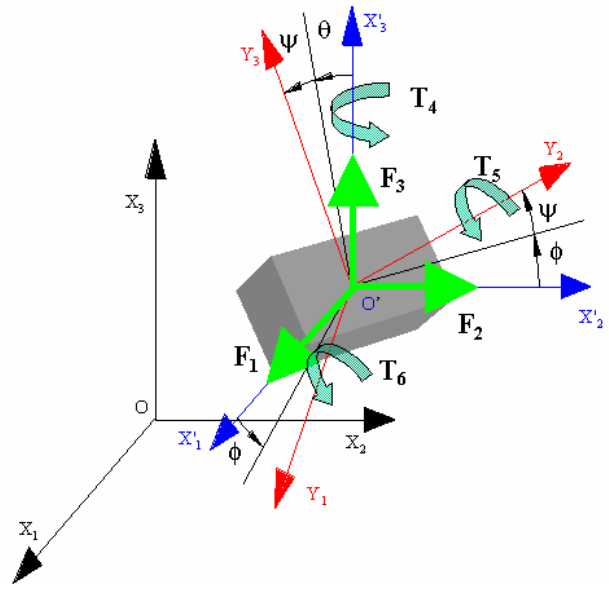

Figure 4. Single body analysis model

The points of start and arriving of links are called nodules. The dynamics of the system can be evaluated using the following process:

1. at the generic instant, position and speed of every nodule can be calculated (starting from the initial conditions);
2. so, considering the deformation and the deformation speed of every link, it's possible to know the force applied by it;
3. so, the forces and torques, which are applied on every mass piece (mass bits an stiff bodies) are known as sum of the actions performed on it by links;
4. for every mass element it's possible to impose the dynamic equilibrium, using the following equation:

$$\|M\| \cdot \|\ddot{X}\| = \sum \|F_i\| \tag{1}$$

where M is the inertial matrix, F is the forces and torques array and $\ddot{X}$ is the array of accelerations.
The code created to implement the automatic calculus receives as inputs some tabs describing the characteristic elastic and viscous curves (in both the axial and transversal directions) of every material which is present in the analysed system. Those tabs relate to the values obtained for pieces of known geometry during the experimental tests.
There are four tabs for every used material giving the dependence force-deformation and force-deformation speed (in the two directions). Considering the generic point of deformation (δ), it's possible to calculate the corresponding force using processes of interpolation (local linear interpolation or spline). When the deformation of one link is known, the applied force can be obtained by those tabs using a shape factor, which refears to the test piece dimensions and to the local link's geometry. For time performance reason a method of temporary linearization in the momentary work point (δ_0) is applied every N steps of integration step (typically every 10 steps): two parameters, depending from the force values given by the explained process, can be calculated for every link (for the elastic force these are called k and $F_0=F(\delta_0)$); so force is given by a simple linear expression, for instance, in the case of the elastic one, it is the one written beside.

$$F(\delta) = k \cdot (\delta - \delta_0) + F(\delta_0) \tag{2}$$

The heat exchange phenomena can be considered, calculating the power exchanged along every single thermal link. That value is given by the following expression:

$$\dot{q}_{A\to B} = \frac{k \cdot (T_A - T_B)}{l} \tag{3}$$

where k is a parameter expressed in [W/mK] which characterises the thermal properties of the used material (for every single link), l is the length of the link and T_A, T_B are the absolute temperatures of two mass elements connected by the considered link. The internal heating, typical of rubber and elastomers, can be evaluated calculating the power dissipated by dampers. So an expression of thermal balance can be written for every mass body and can be integrated in the time dominion.
The purpose of considering the typical non-linearity in temperature of elastomers impose the need to apply a process of double interpolation (in temperature and in deformation, or deformation speed) to know the elastic and viscous forces applied by every link. The software receives in input four tabs (relating to both those forces) for every material at each considered temperature.

5. Results

The dynamic behaviour of the fixture shown in Figure 5 has been simulated using the described software.

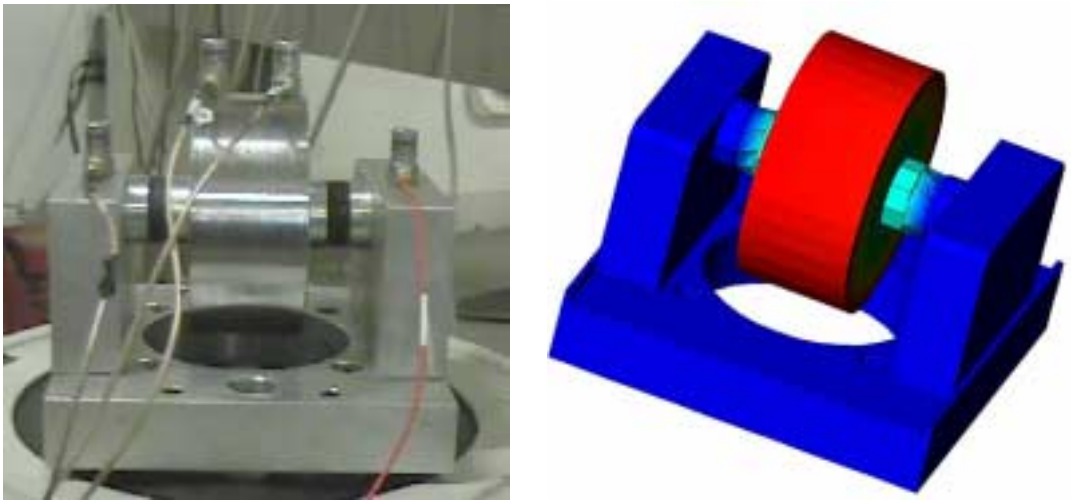

Figure 5. Testing device (here used for numerical simulation)

The model of one of the two rubber cylinders used for the numerical analysis is shown in Figure 6. Every discrete element is characterized by an axial and a transversal stiffness or viscosity, so every rubber band has a stiffness k and every dumper has a dumping coefficient c.

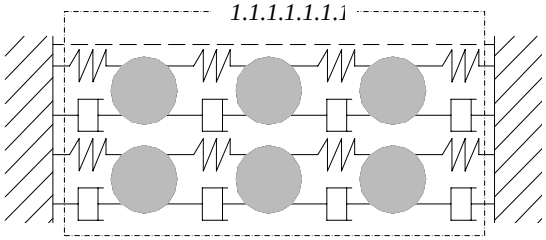

Figure 6. Rubber cylinder model

In this simulation the viscoelastic model used for both the rubber elements is linear, so k and c are constant. Wherever in many different cases k and c are deformation, deformation speed and temperature function as previously shown. The graph of Figure 7 shows the comparison between the simulated results and the experimental data. In this case the resonance frequency is well pointed out by the program, and also the peak amplitude is almost correct. This could be explained, saying that in this second application the viscosity coefficients(c for axial , ct for transversal deformation) have been well evalued.

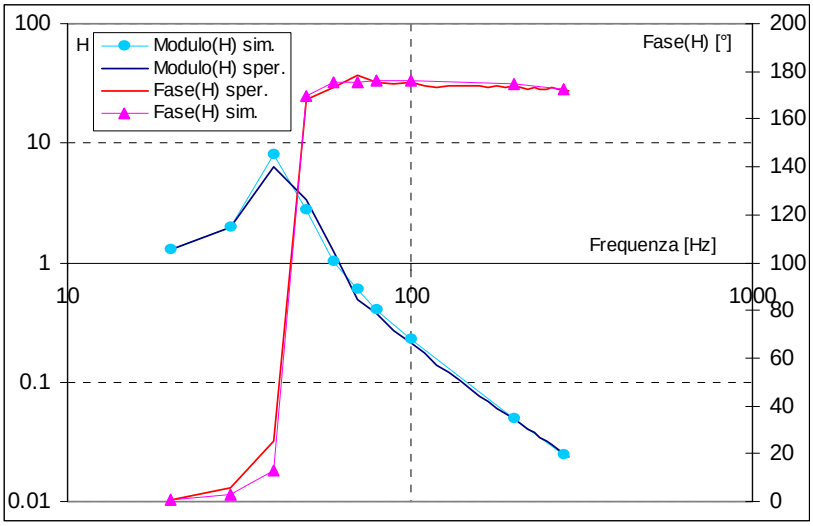

Figure 7. Experimental and numerical results

6. Conclusion

A numerical analysis approach for visco-elastic thermosensible devices has been proposed. This method allows to consider once the effect of non-linearity (strain-force material function) and the effect of temperature variation (reduction of material stiffness) during devices work.
Numerical model parameters (rubber dynamic stiffness and loss-factor) are evalued through an experimental testing of the material used for creating the final device.
Analysis results show that the numerical approach gives good results in comparison with the experimental data.

Acknowledgement

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