

6 ■ OPTIMIZATION OF A VEHICLE A-PILLAR IN A ROLLOVER CRASH

Rajesh Paranjape*, **Sebastian Bawab⁺**, **Resit Unal[†]**, **Gene Hou[§]**
and Stacie I. Ringleb[‡]

**Department of Mechanical Engineering, Old Dominion University, Norfolk, Va, USA 23529.
Tel: +001-7576833244, Fax: +001-7576835344. E-mail: rparanja@odu.edu*

*⁺Department of Mechanical Engineering, Old Dominion University, Norfolk, Va, USA 23529.
Tel: +001-7576833244, Fax: +001-7576835344. E-mail: sbawab@odu.edu*

[†]Department of Engineering Management and Systems Engineering, Old Dominion University, Norfolk, Va, USA 23529. Tel: +001-7576834554, Fax: +001-7576835640. E-mail: runal@odu.edu

*[§]Department of Mechanical Engineering, Old Dominion University, Norfolk, Va, USA 23529.
Tel: +001-7576833728, Fax: +001-7576835344. E-mail: ghou@odu.edu*

*[‡]Department of Mechanical Engineering, Old Dominion University, Norfolk, Va, USA 23529.
Tel: +001-7576835934, Fax: +001-7576835344. E-mail: sringleb@odu.edu*

A response surface method is used in this paper to optimize the structural dimensions of a vehicular A-pillar considered under a rollover crash impact. The objective is to minimize the weight, while the constraints include the yield stress and the maximum allowable deflection. Different cross sections of an A-pillar were subjected to a NHTSA standard impact test. The impact test was simulated using mathematical dynamic model (MADYMO) software by TNO Automotive Safety Systems (TASS), Netherlands. Latin hypercube method was used to generate the sample points. A total of 125 simulations were generated to capture the complete spectrum of three design variables with five levels for each design variable. Response surfaces were obtained by performing regression analysis of the weight, the maximum stress, and displacement in the A-pillar obtained from the results of the simulations. Linear, quadratic, and cubic functions were fitted for the stress, the weight, and the displacement. The best fit was chosen based upon the R^2 and the p-value of the regression analysis. Optimization was then performed to minimize the weight of the A-pillar subjected to the limiting conditions of the stress as well as the maximum allowable displacement. The optimum set of variables obtained was used for the A-pillar in MADYMO to generate the stress and displacement results. These results produced the optimal solution compared to all 125 original sets of data and closely match the solution obtained from the design of experiments technique.

Keywords: A-pillar, Rollover, Design of Experiments, Optimization, Response Surface Method.

1. INTRODUCTION

Vehicle rollover is considered to be one of the leading causes of occupant injury during a crash. According to the National Highway Traffic Safety Administration (NHTSA), sport utility vehicle (SUV) rollover accidents cause the death of one out of every four people who die in auto crashes. Approximately 25% of new cars sold in the United States are SUVs. In 1998, 10,280 fatal crashes involved rollovers whereas in 2002, more than 10,000 people died in rollover crashes. This is typically attributed to a high center of gravity relative to the vehicle's wheelbase. Rollovers are dangerous incidents and have a higher fatality rate compared to other kinds of crashes. Of the nearly eleven million passenger car, SUV, pickup, and van crashes in 2002, only 3% involved a rollover. However, rollovers accounted for nearly 33% of all deaths from passenger vehicle crashes.¹⁻³

The main purpose of this paper was to investigate the effectiveness of the A-pillar in rollover crashes and to optimize the shape of the A-pillar to minimize occupant injury.

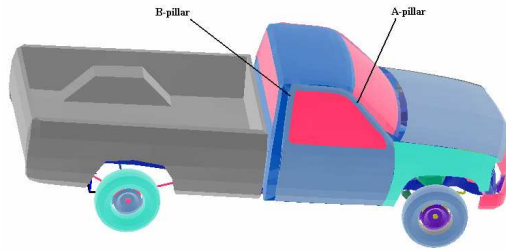


Figure 1. Roof and support structure of a typical light truck.

Figure 1 shows the relative positioning of the roof and its support structure in a typical light truck.

In a typical light truck, there are two structural pillars (labeled A and B). The strength of the A and B-pillars influences the severity of the injuries to the humans involved in the crashes. In a rollover crash, the B-pillar takes heavy loads and is susceptible to most adverse loadings. But, the strength of the A-pillars along with the B-pillars in many cases decides the severity of the injuries to the humans involved in the crashes. Much research has been done to the B-pillar in rollovers compared to the A-pillar prompting us further to investigate and optimize the structural integrity of the A-pillar.^{4–7} Improvements in these support structures can lead to drastic improvement in human safety in a rollover crash.

Different cross sections of A-pillars were subjected to the standard impact test. The impact tests were simulated using mathematical dynamic model (MADYMO) software by TNO Automotive Safety Systems (TASS), Netherlands. For large-scale problems like crash analysis, latin hypercube sampling method is generally employed as a metamodeling technique.^{8–11} We decided to have a grid of five sample points along each variable. As there were three design variables under consideration, a total of 125 (5^3) simulations were generated to capture the complete spectrum of three design variables. The stress and displacement results obtained through the MADYMO simulation were used as constraints to optimize the weight.

The dimensions of the A-pillar obtained from the optimal solution were further simulated in MADYMO to confirm the optimal solution when compared to the original 125 solutions.

2. METHODOLOGY

The A-pillar cross section is modeled as a hollow triangular section as this standard and simple shape best fits a vehicle A-pillar cross section. The optimization of the cross-sectional dimensions of the A-pillar was accomplished by subjecting different cross sections of the A-pillar to a standard impact test, simulated in MADYMO. The Latin hypercube sampling method was employed to solve this problem.^{8–12} There were three design variables under consideration, two for the lengths of the cross section of A-pillar, and one for the wall thickness of the A-pillar. A grid of five sample points along each variable, for a total of 125 simulations, were generated to capture the complete spectrum of the three design variables. The output data obtained through the simulations were used to generate the response surfaces for weight, maximum stress and maximum displacement of each A-pillar. Optimal weight was established under the constraints of stress and displacement.

The accuracy of the solution was confirmed by generating the simulation at the optimal design values, and the output data was compared with the predicted data. The cross section of the A-pillar is assumed as shown in Figure 2.

The total length of the A-pillar was 525 mm. Sides B_1 and B_2 were set as the variables. The angle between B_1 and B_2 was set fixed at 60° . The A-pillar walls were modeled as 2D shell elements with the thickness B_3 considered as the third design variable. The limits of the three design variables are listed in Table 1.

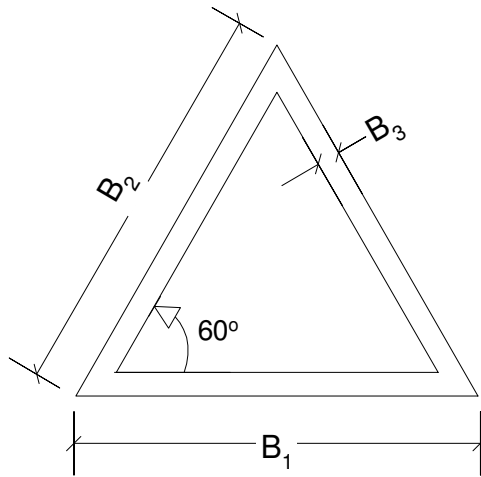


Figure 2. A-Pillar cross section.

Table 1. Limits of the design variables.

Design Variable	Lower Limit (mm)	Upper Limit (mm)
B ₁	60	80
B ₂	50	80
B ₃	3	5

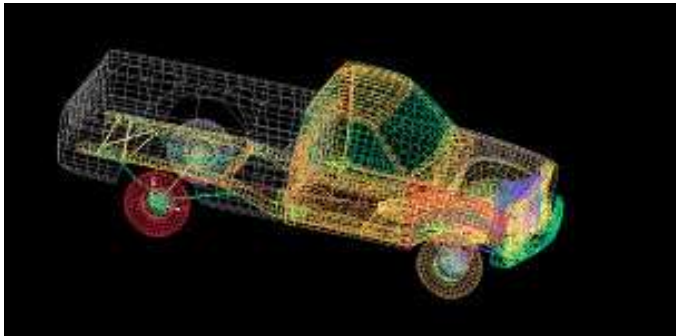


Figure 3. FE model of the truck.

A custom FORTRAN code was used to generate the finite element (FE) model of the A-pillar representing each sample. A total of 126 nodes and 120 shell elements were used in the generation of the A-pillar. This A-pillar was then welded to the fuselage of a finite element Ford F-150 truck. Only relevant parts of the FE truck, such as the cabin, were integrated with the A-pillars. The remaining parts of the truck were converted into a lumped mass and placed at the center of gravity of the truck. This was done primarily to reduce the computation time. Figure 3 shows the complete FE model of the truck. Figure 4 shows the inverted truck cabin with the A-pillars under study and Figure 5 shows the inverted truck cabin with the A-pillars and lumped mass.

The truck was dropped freely with an initial velocity of 7.5 mph when the structure was about to come in contact with the ground as per the NHTSA FMVSS216 guidelines. The total simulation

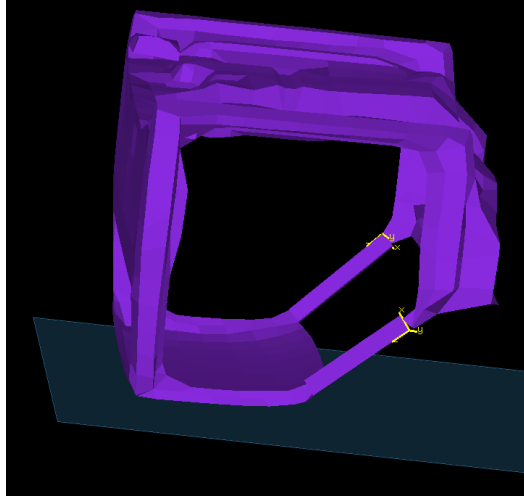


Figure 4. Truck cabin with a-pillars.

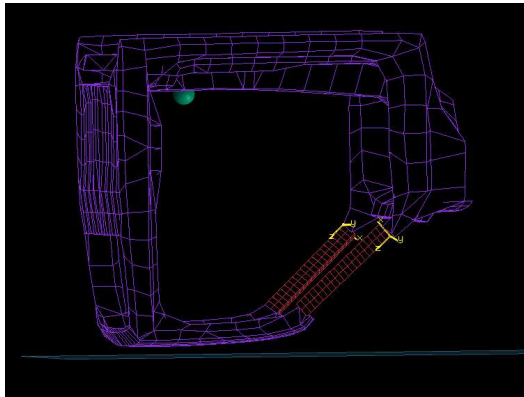


Figure 5. Truck cabin with a-pillars and Lumped Mass.

time is set to 55 milliseconds. During the simulation, von Mises stresses and resultant deflections were reported at each time step. The maximum von Mises stress and the maximum deflection was computed for each sample using a MATLAB code.

The optimization problem in hand could be formulated as:

$$\text{Minimize } \frac{(b)}{\underline{b} \in R^3} \tag{1}$$

Subject to

$$\text{Max}\sigma(\underline{x}, \underline{b}, t) \leq \sigma_0, \underline{b} \in R^3, \underline{x} \in \Omega_{\text{Column}} \tag{2}$$

and

$$\text{Max}\delta(\underline{x}, \underline{b}, t) \leq \delta_0, \underline{b} \in R^3, \underline{x} \in \Omega_{\text{Column}} \tag{3}$$

where

- $w(b)$ = weight of the A-pillar in kg;
- σ = von Mises stress in N/m^2 ;
- σ_0 = permissible von Mises stress in N/m^2 ;
- δ = resultant deflection in meters;
- δ_0 = permissible resultant deflection in meters;
- \underline{x} = position of the node on the column;
- t = time instance during the simulation when stress and deflection is reported; and
- \underline{b} = vector of the three design variables b_1 , b_2 and b_3 .

The range of the three design variables was:

$$0.060 \leq b_1 \leq 0.080 \tag{4}$$

$$0.060 \leq b_2 \leq 0.080 \tag{5}$$

$$0.060 \leq b_3 \leq 0.080 \tag{6}$$

3. DESIGN OF EXPERIMENTS METHOD

Design of experiments is a methodology to achieve a predictive knowledge of a complex, multi-variable system with the fewest trials possible. Due to the possibly complex nature of the response surfaces, five levels for three-variable study were designed as illustrated in Table 2 and the simulations were run at those discrete points over the sample space.^{13–15}

After running all 125 simulations in MADYMO, data at different time intervals were obtained. A custom MATLAB program was used to report the maximum stress and displacement over the A-pillar during each simulation. The weight of the A-pillar in each simulation was determined using material properties of steel with a density of $7.89E+03 \text{ kg/m}^3$.

Least square regression analysis was performed on the data with linear, second-order, and third-order approximation models for weight, stress, and displacement. Equations (7) and (8) represent the second-order mathematical model for weight and displacement respectively. Equation (9) represents third-order mathematical model for stress. The second-order approximation model fit well for the response surface for the weight as well as the displacement as illustrated in Equations (7a) and (8a), respectively. A third-order approximation model best fit the response surface for the stress as indicated in Equation (9a).

Table 2. Design Parameter Matrix.

Run #	B_1	B_2	B_3	B_1^2	B_2^2	B_3^2	B_1B_2	B_1B_3	B_2B_3	B_1^3	B_2^3	B_3^3
1	-2	-2	-2	4	4	4	4	4	4	-8	-8	-8
2	-1	-2	-2	1	4	4	2	2	4	-1	-8	-8
3	0	-2	-2	0	4	4	0	0	4	0	-8	-8
4	1	-2	-2	1	4	4	-2	-2	4	1	-8	-8
5	2	-2	-2	4	4	4	-4	-4	4	8	-8	-8
6	-2	-1	-2	4	1	4	2	4	2	-8	-1	-8
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
121	-2	1	2	4	1	4	-2	-4	2	-8	1	8
122	-1	1	2	1	1	4	-1	-2	2	-1	1	8
123	0	1	2	0	1	4	0	0	2	0	1	8
124	1	1	2	1	1	4	1	2	2	1	1	8
125	2	1	2	4	1	4	2	4	2	8	1	8

$$Weight = c_0 + \sum_{i=1}^k c_i b_i + \sum_{i=1}^k c_{ii} b_i^2 + \sum_{i<j}^k \sum_{i<j} c_{ij} b_i b_j \quad (7)$$

$$\begin{aligned} Weight = & 3.3035 + 0.1482 * b_1 + 0.1351 * b_2 + 0.3634 * b_3 \\ & + 0.001731 * b_1^2 + 0.000546 * b_2^2 - 0.00554 * b_3^2 \\ & - 0.00218 * b_1 * b_2 + 0.0179 * b_1 * b_3 + 0.0177 * b_2 * b_3 \end{aligned} \quad (7a)$$

$$Displacement = c_0 + \sum_{i=1}^k c_i b_i + \sum_{i=1}^k c_{ii} b_i^2 + \sum_{i<j}^k \sum_{i<j} c_{ij} b_i b_j \quad (8)$$

$$\begin{aligned} Displacement = & 0.100305 - 0.00655 * b_1 + 0.000317 * b_2 - 0.00597 * b_3 \\ & + 0.000398 * b_1^2 + 0.000285 * b_2^2 - 0.000163 * b_3^2 \\ & - 0.00042 * b_1 * b_2 - 0.0003 * b_1 * b_3 + 0.0000536 * b_2 * b_3 \end{aligned} \quad (8a)$$

$$Stress = c_0 + \sum_{i=1}^k c_i b_i + \sum_{i=1}^k c_{ii} b_i^2 + \sum_{i=1}^k c_{iii} b_i^3 + \sum_{i<j}^k \sum_{i<j} c_{ij} b_i b_j \quad (9)$$

$$\begin{aligned} Stress = & 2.86e8 - 230746 * b_1 - 162764 * b_2 - 1449373 * b_3 \\ & - 104690 * b_1^2 + 20106 * b_2^2 + 158816 * b_3^2 - 27154.7 * b_3^3 \\ & + 93708 * b_1 * b_2 - 109380 * b_1 * b_3 + 130750.8 * b_2 * b_3 \end{aligned} \quad (9a)$$

In Equations (7), (8), and (9), b_i terms are the input design variables that influence the response, weight, displacement, and stress respectively. And c_0 , c_i , c_{ii} , and c_{ij} are the estimated regression coefficients. The cross terms represent two-parameter interactions. The square and cubic terms indicate non-linearity.

Table 3 displays the regression analysis results for the A-pillar weight. The model fit was excellent in this case with an indicated adjusted R^2 value of 0.999525.

Table 4 displays the regression analysis results for the maximum allowable displacement. The model fit was excellent in this case with an indicated adjusted R^2 value of 0.99451.

Table 5 displays the regression analysis results for the stress. The model fit also was excellent in this case with an indicated adjusted R^2 value of 0.993038.

Once response surface equations are developed, they can be used to determine the effect of varying design-variable values on the response characteristics and the optimization of the model becomes more reliable and efficient. The regression models were formulated for the weight, displacement, and stress as indicated in Equations 7a, 8a, and 9a respectively. The weight of the A-pillar was optimized

Table 3. Regression statistics: Weight model.

Regression Parameters	Value
R^2	0.99956
Adjusted R^2	0.999525
Standard Error	0.012893

Table 4. Regression statistics: Displacement model.

Regression Parameters	Value
R^2	0.994909
Adjusted R^2	0.99451
Standard Error	0.000941

Table 5. Regression statistics: Stress model.

Regression Parameters	Value
R ²	0.993712
Adjusted R ²	0.993038
Standard Error	190571.2

Table 6. Optimal solution using solver.

Design Variable	Solver Solution
B ₁	2
B ₂	-2
B ₃	-1.3129109

Table 7. Optimal solution.

Design Variable	Solver Solution (mm)
B ₁	80
B ₂	55
B ₃	3.345

Table 8. Comparison of optimal solution with MADYMO simulation.

Design Variable	Solver solution	MADYMO Solution	Difference
Weight	2.8602 kg	2.9695 kg	3.82%
Stress	2.88E+08 N/m ²	2.87E+08 N/m ²	0.49%
Displacement	0.1 m	0.0989 m	1.1007 %

subjected to the displacement and yield stress constraints over the sample space of the three design variables B₁, B₂, and B₃. The optimal solution obtained through the solver is listed in Table 6.

In the design of experiments, (-2) represents the minimum value of the design variable and (+2) represents the maximum value of the design variable. The transition from (-2) to (+2) is linear. The solver solution is denormalized to obtain the exact physical dimensions of each design variable at the optimal point. These values are listed in Table 7.

The accuracy of the solution was validated by running a MADYMO simulation with the design variables for the A-pillar mentioned in Table 7. The results obtained through the MADYMO simulation were compared to the solver solution as shown in Table 8. The difference between the weight obtained from the solver and through the MADYMO simulation was less than 4% whereas the difference between the maximum stress obtained through the solver and the MADYMO simulation was less than 0.5%. In addition, the difference between the maximum allowable displacement obtained through the solver and the MADYMO simulation was about 1.1%. It is evident from Table 8 that the optimal solution achieved through the solver matches very closely to the simulated solution.

4. CONCLUSION

This paper focused on optimizing the cross section of a vehicular A-pillar considered under a rollover crash. Design of experiments methodology was used to set up the design space. Response surface methodology was used to generate a close relationship between the desired responses (weight, stress, and displacement) and the design variables (B₁, B₂, and B₃). Optimization was used to find the minimum weight of the A-pillar under the limiting stress and maximum allowable displacement constraints

over the design space. The efficiency and accuracy of the overall methodology was established using the confirmatory MADYMO simulation at the optimal solution.

Design of experiments and response surface methodology offer a systematic approach of studying the parameter space for model building and design optimization. The results obtained from MADYMO simulation indicate that the design of experiments method along with the regression analysis and optimization provided an accurate optimal solution. Furthermore, design of experiments technique can provide a reliable optimal solution for complex problems such as crash analysis. This technique can lead to savings in computational, design, and material costs that in turn can provide a more robust as well as cost-effective product.

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