Evaluation of linear drives with different fuzzy methods

Attila Piros und Tibor Bercsey
Department of Machine and Product Design
Budapest University of Technology and Economics

This paper focuses on the selection of the mechanical components of a linear drive system. The system itself is installed in a high capacity tool magazine. The high variety of the possible components makes the selection procedure complicated because of the large number of the possible variations. The manual handling of this procedure, like managing the data with spreadsheets, is nearly impossible. Using fuzzy membership functions to evaluate the mechanical properties of these components gives an opportunity to automatically find the best combination of the components. There are many fuzzy evaluating methods available, but this paper compares the conventional fuzzy evaluation (R_G, R_{FZ}) with using corrected fuzzy mean (CFZ).

1 Setting up the basic fuzzy membership functions

The fuzzy method itself is based on the evaluation of the calculated properties of the selected set using fuzzy logic [1]. This method uses fuzzy membership functions to describe the properties of the components. The fuzzy logic gives an opportunity both for handling the human point of view and the ambiguous cases of the evaluation of variations [2]. This way the kind of notions like temporal over load of the servo motor can be interpreted.

In the selection procedure the following components are varied in case of three robot motion axes [3]: linear guide with drive mechanism (5 types), gearbox (25 types), coupling (9 types), servo motor (11 types). Because of

the large number of possible combinations compatibility functions were applied to validate the combinations at the first stage. These compatibility functions pre-filter the selected sets of components. For example if a specific combination of the motor/gearbox has incompatible axes diameters then this variation is dropped before the further calculations. After the application of the compatibility functions only 1519 are left from 37125 variations for further process. The software generates all possible combinations among the components and also calculates the following compatibility values listed in Table 1.

Table 1: List of compatibility functions

ID	Compatibility test between these values:
CP01	Motor/Coupling shaft diameter.
CP02	Motor/Coupling torque in case of acceleration.
CP03	Motor/Gearbox torque value.
CP04	Coupling/Gearbox type.
CP05	Gearbox/Load torque value.
CP06	Gearbox/Guide ratio.
CP07	Guide/Axis type.

Some mechanical properties of the kinematic chain must be calculated and evaluated to find the appropriate combination of the selected components. First the required torque value (T_{CL}) is calculated at the load side then this value is recalculated to the servo motor side (T_{CM}) with the following functions:

$$T_{CL} = m_L \cdot g$$

$$T_{CM} = \frac{T_{CL}}{i_{GB} \cdot \eta} + T_{frM}$$

m_L: mass of load
g: gravity const.
i_{GB}: gearbox ratio
η_{GB}: gearbox efficiency
T_{frM}: friction torque of motor

Knowing the kinematic properties the reflected load inertia (I_{RL}) and the inertia ratio at the motor side (R_{IL}) have to be calculated:

$$I_{RL} = \frac{m_L \cdot DPR_G{}^2}{2 \cdot \pi} + I_{GB} + I_C$$

$$R_{IL} = \frac{I_{RL}}{I_M}$$

DPR_G: guide ratio
I_{GB}: gearbox inertia
I_C: coupling inertia
I_M: motor inertia

Concerning the required acceleration torque at the load side (T_{aL}) and the gearbox ratio the total acceleration torque at the motor side (T_{aM}) can be calculated with these functions:

$$T_{aL} = v_M \cdot \frac{2 \cdot \pi}{t_a} \cdot I_M + \frac{I_{RL}}{\eta_{GB} \cdot \eta_G} \qquad v_M: motor\ speed$$
$$T_{aM} = T_{CL} + \frac{T_{aL}}{i_{GB}} \qquad \begin{matrix} t_a: acceleration\ time \\ \eta_G: guide\ efficiency \end{matrix}$$

The final results of these calculations are listed in Table 2. These values are the base of the Fuzzy evaluation.

Table 2: The variant properties in the design process

Name	Description
speed (SP)	Speed of the moving load at the end of the kinematic chain.
inertia ratio (IR)	Ratio between the reflected load inertia and the motor inertia.
maximum torque (TM)	The required torque at the motor shaft in case of acceleration.
stall torque (TS)	The required torque at the motor shaft in case of constant velocity.
utilization ratio (UR)	Ratio between the motor maximum torque and the required acceleration torque.

In this stage all of the possible configurations are automatically generated to cover the whole design space. Generating means that the components are only paired without any tests. However, this generation procedure is quite fast, further evaluation requires many more related calculations. Using compatibility functions the number of possible valid sets are significantly decreased.

The values of the mechanical properties are the base of the Fuzzy evaluation. Figure 1 shows the fuzzy membership functions for these properties.
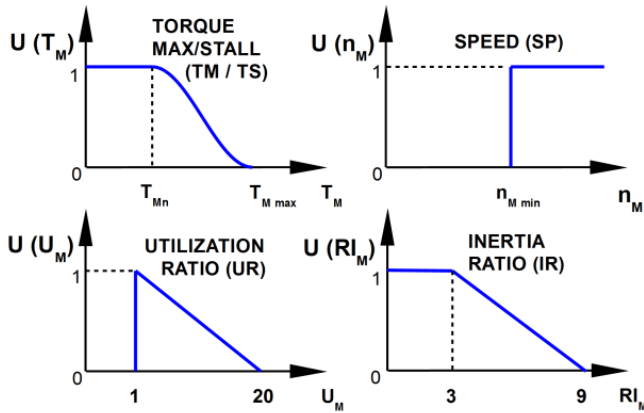
Figure 1: Fuzzy membership functions of the variant properties

The results are organized into a 6 dimensional matrix. By fixing 3 coordinates there is a good opportunity to visualize a 3-dimensional subset from this matrix. Figure 2 shows the selected portion of the matrix with the different fuzzy values and the incompatible combinations in colour yellow. Here the robot axis (X), the linear guide type (THK GP8-20C) and the coupling (ATEK GS24 KN) are fixed. The servo motor type and the gearbox are the varying components. The different mechanical properties are displayed on the vertical axis.
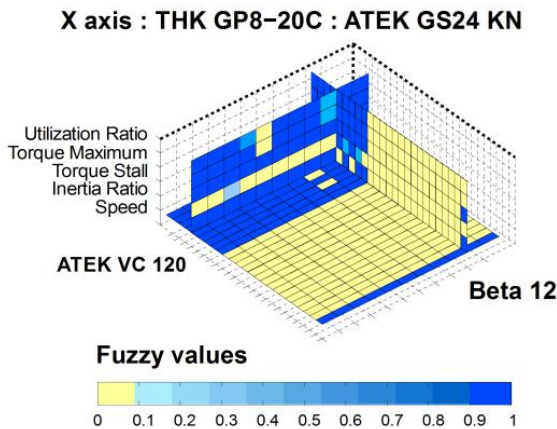

Figure 2: 3-dimensional subset of the 6 dimensional solution space

2 Conventional fuzzy evaluation of the results

The conventional fuzzy evaluation is based on the fuzzy logic. This logic can handle inaccurate data and it can also model nonlinear functions of arbitrary complexity. With Fuzzy Inference Systems (FIS) the mapping of input data onto the output space can be made in a very convenient way. In the current case, on one side the FIS contains the parameters' fuzzy membership functions for evaluating the input data (the mechanical properties of the given set of components). On the other side this system also includes the output fuzzy membership functions of the quality (Q) of the components' set.

The working of FIS must meet two fundamental criteria (C1 C2). The first criteria says that if any fuzzy value is equal to 0 the quality value must be 0. In this case this variation is out of the range because of the failure of one of its parameters. The second criteria concerns the ideal case. If all of the parameter values are equal to 1 the calculated quality must also be 1 (1 marks the optimal value).

$$C1: 0 \in U_i \rightarrow Q_i = 0$$
$$C2: \frac{\sum_{i=1}^{n} U_i}{n} = 1 \rightarrow Q_i = 1$$

The working of FIS is based on fuzzy if-then rules, simply, fuzzy rules. In this case two rules are required to fulfil the two basic criteria (

$$1: \neg input1 \vee \neg input2 \rightarrow output = bad \quad 2: input1 \wedge input2 \rightarrow output = good$$

On the next illustration the inertia ratio (IR) and the maximum torque (TM) are displayed as the input functions. As the output function the Quality is pictured in Figure 3. This image shows the case of criteria C1, when one of the input parameters (TM) fails therefore the value of the output function is also 0.
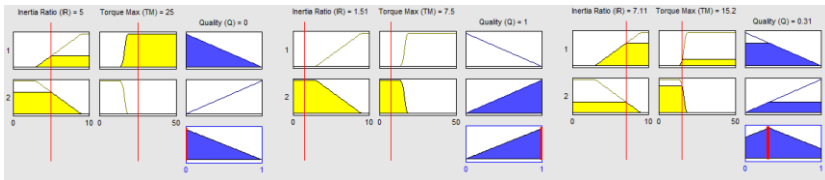

Figure 3: The worst, the optimal and the intermediate case of the parameters

This figure also shows case two (C2), when all of the parameters are ideal and the output value is also 1.

In an intermediate case the output function results a value between 0 and 1 as Figure 3 shows it. As this picture well displays the week value of one input (Inertia Ratio) significantly decreases the output (Quality) value. This feature of the conventional fuzzy evaluation will be quite important in the interpreting the results of the comparison.

On all of the images the negated input functions (in yellow colour) are displayed in the top row together with the "bad" output function of quality (in blue colour). The direct input functions (from left the Inertia Ratio (IR) followed by the Torque Max (TM) function) are in the middle row together with the "good" output function. The unified output function is displayed at the bottom-right corner of the images.

In the evaluation of a specific set of components all pairing of the mechanical properties are studied with the concerning FIS (see in Table 3.).

Table 3: The properties and the paired (FIS) functions

		1	2	3	4	5
		SP	**IR**	**TS**	**TM**	**UR**
1	**SP**	1	(SP,IR)	(SP,TS)	(SP,TM)	(SP,UR)
2	**IR**	1	1	(IR,TS)	(IR,TM)	(IR,UR)
3	**TS**	1	1	1	(TS,TM)	(TS,UR)
4	**TM**	1	1	1	1	(TM,UR)
5	**UR**	1	1	1	1	1

The bracketed FIS functions, like the previously described Inertia Ratio - Maximum Torque (IR,TM), are weighted with the following formula.

$$(FIS) \cdot \frac{\sum_{i=1}^{n} w_i}{n} \quad w_i: weight\ of\ the\ property$$

Table 3 shows the structure of the output matrix. Initially this matrix is filled with 1 values. During the evaluating procedure the (FIS) output values are calculated in each pairing. Because these values are between 0 and 1 the minimum value of the matrix is easy to find and it is also very significant. The minimum characterizes the whole combination of this drive chain, which is not better than its weakest property.

3 The corrected fuzzy mean

Based on the independently calculated fuzzy values the corrected fuzzy mean (R_{FZ}) is an average to compare the different configurations. This average is similar to the geometric mean (R_G). The weighted formulas of these averages are the following:

$$R_{FZ} = \frac{U_1^{w_1} \cdot U_2^{w_2} \cdot ... \cdot U_n^{w_n}}{\frac{\sum\limits_{i=1}^{n} U_i}{n}}$$

$$R_G = \sqrt[\sum\limits_{i=1}^{n} w_i]{U_1^{w_1} \cdot U_2^{w_2} \cdot ... \cdot U_n^{w_n}}$$

Both means meet the two fundamental criteria (C1 C2). The main difference between these means is the distribution of the mean values. The values of the geometric mean are distributed on a narrower range than the values of the corrected fuzzy mean. The broader distribution range of the calculated mean values makes the evaluation much easier in case of corrected fuzzy mean. This range is more than two times wilder in the corrected fuzzy mean than the geometric mean which is a big advantage in the software algorithm. Figure 4 shows that the corrected fuzzy mean separates the design variations better than the geometric mean.

4 Comparison of the fuzzy evaluations

The two main strategies, the conventional fuzzy evaluation and averaging the independent fuzzy values have numerous differences. As Figure 4 clearly displays the conventional fuzzy evaluation (CFZ in green) distributes the values as wilder than the corrected fuzzy mean (R_{FZ} in magenta) and the characteristics of the three diagrams (together with the geometric mean (R_G in blue) are very similar. But the CFZ values (plotted with green dots) are originally ordered in a different way than the two means. Generally it would be problematic, but the aim of the selection procedure is to find best solutions. In the current case the first 20% of the best solution is in the same order in all of the evaluating methods.
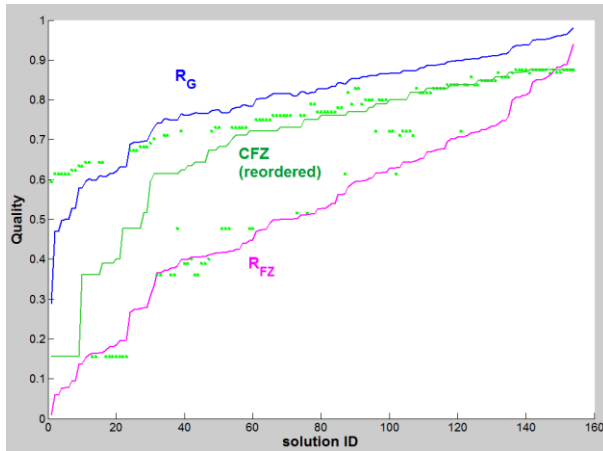
Figure 4: Results of the 3 evaluating methods

5 Conclusions

This paper clearly shows that the fuzzy based evaluation makes the selection procedure faster and more perspicuous. Unlike the manually executed calculations the software covers the entire solution space. It is also quite obvious that the calculation of the corrected fuzzy mean is much faster than generating and using high number of the Fuzzy Inference Systems. In the software algorithms the application of the corrected fuzzy mean has a notable advantage in the evaluation of the results. This method distributes the results in the broadest range.

References

[1] Deciu, E. R. , Ostrosi, E. , Ferney, M. and Gheorghe, M. "Configurable product design using multiple fuzzy models", Journal of Engineering Design, Vol. 16 No: 2 (2005), pp 209 - 233

[2] Dombi, J., Gera, Zs., "Rule based fuzzy classification using squashing functions", Journal of Intelligent & Fuzzy Systems: Applications in Engineering and Technology, Vol. 19 , No: 1 (2008), pp 3-8

[3] Piros, A. "Fuzzy Evaluation in Configurable Product Design", The 2nd INTERNATIONAL CONFERENCE ON DESIGN ENGINEERING AND SCIENCE, Tokyo, JAPAN, November 17-19, 2010