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TOWARD A CONFIGURATION DESIGN METHOD FOR MECHANICAL METAMATERIALS

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ABSTRACT
Metamaterials are materials that exhibit behavior that is not observed commonly in nature, but that can be achieved by designing often complex geometric structures. Behaviors such as negative index of refraction, negative Poisson's ratio, or negative stiffness can be achieved with suitable lattice or other cellular structures at appropriate size scales. Such materials can be used in many applications to achieve functionalities that would otherwise be difficult to realize and can lead to innovative solutions. Assuming they exist, designers need a design method capable of identifying when metamaterials are suitable to use as a component in a developing system design. In this paper, we present a configuration design method to meet this need. From a selected function, designers should identify a suitable physical principle and model of desired behavior, then these can be matched to metamaterials and their behavior capabilities. Examples are presented using mechanical metamaterials including bistable mechanisms that exhibit negative stiffness and lattice structures that exhibit negative Poisson's ratio.

Keywords: Metamaterials, bistable mechanism, negative Poisson's ratio, configuration design

1 INTRODUCTION

Metamaterials are materials that exhibit some behavior that is not observed commonly in nature, such as negative index of refraction, negative Poisson's ratio, or negative stiffness [1]. Designers can take advantage of such behaviors by designing cloaks, for example, that can "hide" objects from detection by electromagnetic, vibration, acoustic, or other energy. Such cloaking devices act as band-gap filters, taking advantage of a negative index of refraction for light or negative Poisson's ratio for vibrations. Many other types of applications have exploited such metamaterials, some of which will be explored in this paper.

Metamaterials are not homogeneous, solid blocks of metal or polymer; rather they are typically geometric structures or compositions of multiple bulk materials. Their behaviors are the result of a combination of the properties of their constituent bulk materials as well as the designed structuring. For example, structures with negative Poisson's ratio behavior are typically specially shaped lattice structures [2] as shown in Figure 1. Structures for cloaking of light or sound have other, more complex geometric shapes, but their presentation is out of scope for this paper.

In the metamaterials literature, the design of devices with metamaterial is largely ad hoc and focused on the mathematical models governing the metamaterial behavior of interest. It is unclear if any attempt has been made to systematize design methods and develop a more general design method for any specific metamaterial type, much less the class of metamaterials. This paper is an attempt to offer insights into modeling and designing metamaterial devices that could lead to a comprehensive design method. That is, the purpose of this paper is to help designers identify when metamaterials should be used in the design of an engineering system, rather than the design of the metamaterial itself. We focus on mechanical metamaterials with negative Poisson's ratio and that exhibit intermittent motions, in the form of bistable mechanisms [3], since they correspond to our areas of interest and experience. Reasoning about their unique behaviors is central to the method; however, we demonstrate that this reasoning can be systematized and embedded into an iterative design method by focusing on the device's forces, displacements, and energy relationships.

By configuration design, we mean the identification of components and devices that are in the designed system, their arrangement spatially and logically, and their interactions with other parts of the system [4]. This topic is highly related to product architecture design [5] where the emphasis is on establishing the relationship between functions and forms. In this work, we focus on establishing an initial solution structure by identifying an appropriate metamaterial, but do not go further in analyzing modules and refining modular architectures using design structure matrix techniques, as many other researchers do (e.g. [6]. We start with a function structure and develop a behavior model, then select appropriate structures, borrowing heavily from function-behavior-structure [7] literature. Furthermore, we include some qualitative reasoning of behavior, much like the structure-behavior-function method [8]. A fully developed design method for systems that utilize metamaterials will likely share characteristics with other computational design synthesis methods such as those proposed in [9, 10]. At this preliminary stage, however, the proposed design method is of narrow scope and is focused on identifying when metamaterials are suitable using simplified causal reasoning models.

In the next section, we provide an overview of mechanical metamaterials, explain their behavior, and develop mathematical expressions of their behavior that will be utilized in the design method. The proposed configuration design method is presented in Section 3, followed by two examples in Section 4. Conclusions are drawn and future work outlined in the final section.

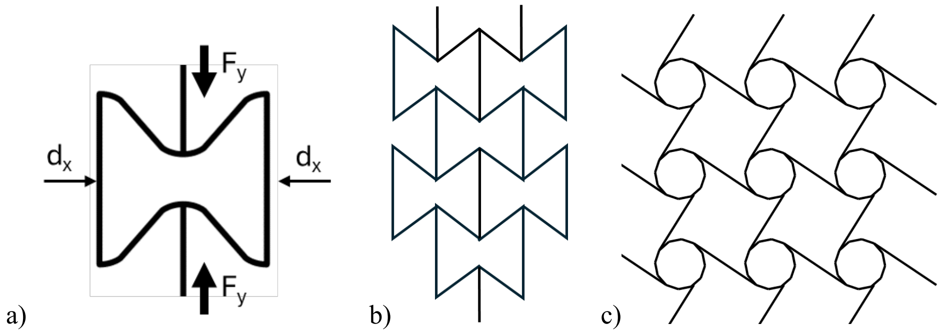


Figure 1: Auxetic materials, a) single unit cell, b) array of unit cells with simplified geometry, c) array of chiral cells. In all cases, applied compressive forces F_y cause contraction motions d_x.

2 MECHANICAL METAMATERIALS

Two types of mechanical metamaterials will be presented here, auxetic structures and bistable mechanisms.

2.1 Auxetic Structures

The term auxetic denotes negative Poisson's ratio. Typically, when a material is stretched, it contracts in lateral directions, consistent with the concept of conservation of mass. However, auxetic materials expand laterally when stretched; similarly they contract when compressed, as illustrated in Figure 1. Cork is an example of a material with approximately 0 Poisson's ratio; but as a general rule, materials have positive Poisson's ratios. Hence, auxetic behavior is achieved by constructing geometric shapes that have the desired behaviors. Two simple constructions are shown in Figure 1, but many more exist (good reviews are provided in [2, 11]).

Considering force-displacement relationships associated with behaviors governed by solid mechanics, a typical material exhibits a positive displacement in the direction of a given force. For the purposes of this presentation, displacements will be considered positive if they are in the same direction as forces that cause them. Hence, if a compressive force F_y in Figure 1a is considered negative, then it causes a negative displacement, $-d_y$. In the lateral direction X, a material with a positive Poisson's ratio will exhibit a positive displacement in the X direction, d_x. In contrast, an auxetic material, such as in Figure 1a, will exhibit a negative displacement. In 3 dimensions, a variety of similar reentrant lattice constructions are common as seen in Figure 2. For 3D, given the same compressive force, $-F_y$, the lateral X and Z displacements, d_x and d_z, respectively, will be negative for an auxetic material.

A behavior model can be derived for auxetic materials using these force and displacement terms. Such models will be given in the form of a rule with causality. The symbol ⇒ denotes "causes" in this work. In general 3D context, the behavior rule can be expressed as

$$F_x > 0 \Rightarrow d_x > 0 \Rightarrow d_y > 0, d_z > 0$$

This rule can be stated as: for a positive force F_x causes a positive displacement d_x which, in turn, causes positive lateral displacements d_y and d_z. Of course, the subscripts (directions) can be switched around to suit the geometric configuration being modeled.
In the case of 0 Poisson's ratio materials (0PR), the lateral displacements will be 0.

2.2 Bistable Mechanisms

A bistable mechanism has two stable equilibria corresponding to two different positions of the links in the mechanism. Transitions between states typically require little net power; input power and energy to cause the transition can be recovered in subsequent motion, which is helpful when the mechanism is used as an actuator. On the other hand, bistable mechanisms can be designed to absorb energy by transitioning between states with a loss of energy. Furthermore, when designed properly, the stable states of the mechanism can be robust to external disturbances which facilitates applications for locking mechanisms and energy traps. Figure 3 shows two types of bistable mechanisms. On the left, a simple mass is suspended by beam attached above and below it. The mass shuttles back and forth between the two positions indicated using solid and dashed graphics. In the left configuration, a force to the right that is above a certain threshold can cause the mass to snap to the dashed configuration. In contrast, forces below the threshold do not cause the transition, indicating stability. On the right, a 4-layer array of bistable mechanisms exhibits progressive contraction through beam buckling [12].
The metamaterial phenomenon exhibited by bistable mechanisms is negative stiffness. This type of behavior can be seen readily by investigating the force-displacement relationship (similarly the stress-strain relationship) of the mechanism as it undergoes loading. As is well known, the slope of a material's stress-strain curve is its stiffness, more formally called the elastic modulus. If the slope becomes downward, this indicates negative stiffness.

Figure 2: 3D auxetic unit cell.

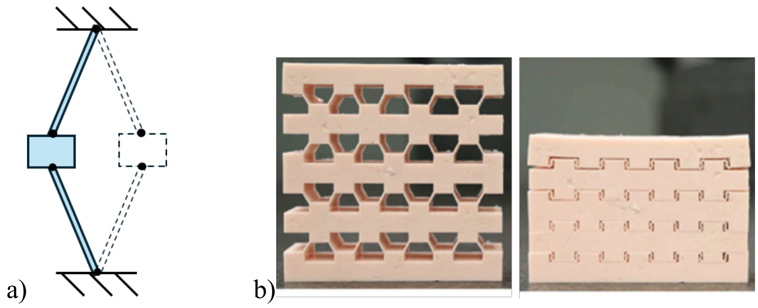


Figure 3:Two types of bistable mechanisms that rely on beam buckling to transition between equilibrium states. Mechanism b) [12] is considered to be multi-stable since it consists of 4 rows of collapsible beams.

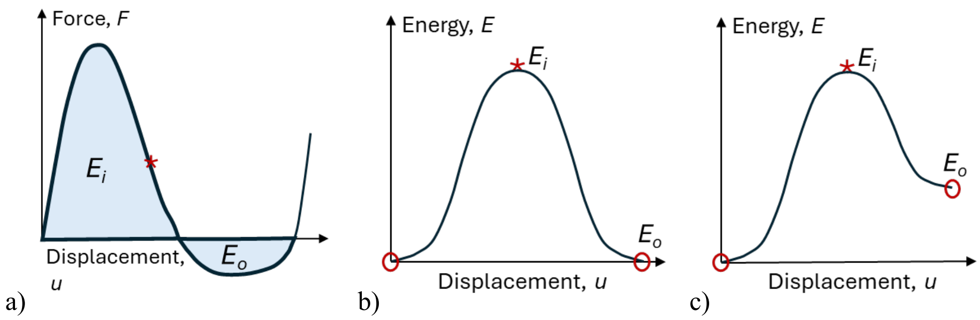


Figure 4: Notional force and strain energy relationships as a function of displacement for the mechanism in Fig. 3a (a, b) and energy for the mechanism in Fig. 3b (c). Red circles indicate stable equilibria. The red '' indicates an unstable equilibrium position.*

A typical force-displacement curve for a bistable mechanism is shown in Figure 4a. After reaching its peak, the curve turns downward, corresponding to the buckling of a beam(s) in the mechanism. If the curve goes into the negative force region, this indicates that the buckled beam is transferring energy to the structure; that is, it is exerting a force in the direction of mechanism motion. The area under the force-displacement curve indicates the strain energy associated with the material. The area labelled E_i

indicates the energy associated with deforming the mechanism until it reaches the snap-through point. Area E_o indicates the energy into the mechanism from the buckled beam. The difference $E_i - E_o$ corresponds to the energy that is "locked into" the mechanism after snap-through; much of this can be recovered upon returning it to its original configuration. Strain energy as a function of displacement is plotted in Figure 4b that shows energy troughs that correspond to the bistable mechanism's two stable states. The peak between them represents the energy barrier that needs to be applied to cause a state transition. This peak corresponds to the inflection point of the force-displacement curve in its down trajectory. These force-displacement and energy phenomena can be applied in many applications, three of which are described here.

Actuator

Qualitatively, if a bistable mechanism is pushed so that it deforms a bit, then it will snap-through, providing a jump in displacement. The ratio d_o / d_i indicates the motion amplification that results, where d_i is the displacement until snap-through and d_o is the displacement after snap-through. Stated mathematically, a force F_i resulting in a displacement d_i causes an increased displacement d_o. Stated as a rule, this relationship could be expressed as

$$F_i > 0 \Rightarrow u_i = d_i \Rightarrow u = d_o$$

where u is the displacement and is in the same direction as F_i.

Energy Absorber

Bistable mechanisms have been proposed for many types of energy absorber applications, including armor, helmets, and other sport equipment. Considering the energy-displacement curve in Figure 3b, energy absorption is indicated by the difference between the peak and the right-hand stable state, $E_i - E_o$. This energy difference is a function of both the snap-through behavior and any hysteresis in the mechanism itself, typically caused by viscoelastic behavior of its material. Stated as a rule, the relationship between these quantities can be expressed as

$$F_i > 0 \Rightarrow u = d_i,\ E = E_i \Rightarrow u = d_o > d_i,\ E_{abs} = E_i - E_o$$

where E indicates strain energy and E_{abs} denotes absorbed energy. Restated, input force F_i causes displacement d_i and strain energy E_i which then causes the jump to displacement d_o and energy absorption E_{abs}.

Contraction/Expansion with Intermittent Motion

This application could be considered as a special case of an actuator, but is intended to indicate that it is the change in length of a structure as being of interest. The behavior can be explained as an applied force F, less than or equal to some force threshold, F_{th}, causes a series of intermittent motions. Expressed as a rule, the behavior can be described by

$$F_{th} > F_i > 0 \Rightarrow u = n\ \delta$$

where n is the number of intermittent motions (indicating the number of bistable mechanisms linked serially) and δ is the magnitude of the displacement.

3 DESIGN LIBRARIES

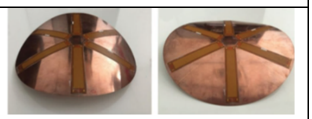
Since awareness is an important element in designing with metamaterials, the development of browsable design libraries is proposed to enable designers to explore different physical principles and metamaterial types and applications. The two design libraries described here include information that will be useful in support of system design. The first design library is for physical principles, as presented in Table 1, which has three representative principles, 1 or 2 levels of sub-principles, and qualitative behavior models similar to those presented in Section 2. The entries were chosen to for their relevance to designing with metamaterials and to the specific mechanical metamaterials of interest in this paper. The insight that led to this library was that simple qualitative behavior models were sufficient to distinguish common physical principles from the uncommon principles that govern the behavior of metamaterials. Regarding notation, most symbols were defined in Section 2. Others are: T is torque, φ is rotation angle, η is refractive index, and θ_x are angles of refraction.

Table 1. Physical principle design library

Solid mechanics	**Sub-topic**	**Qualitative behavior model**
Continuum mechanics	Positive Poisson's ratio	$F = \mathbf{E}d,\ F_i > 0 \Rightarrow d_i > 0 \Rightarrow\ d_j < 0,\ d_k < 0$ $i,j,k \in \{\mathrm{X, Y, Z}\}$
	0 Poisson's ratio	$F_i > 0 \Rightarrow d_i > 0 \Rightarrow\ d_j \cong 0,\ d_k \cong 0$
	Negative Poisson's ratio	$F_i > 0 \Rightarrow d_i > 0 \Rightarrow\ d_j > 0,\ d_k > 0$
	Positive stiffness	$F_j > F_i > 0 \Rightarrow d_j > d_i > 0$
	Negative stiffness	$F_j < F_i > 0 \Rightarrow d_j > d_i > 0$
Kinematics		
Intermittent motion	Linear motion (linear ratchet)	$F_{th} > F_i > 0 \Rightarrow u = n\ \delta$
	Rotary motions (rotational ratchet)	$T_{th} > T_i > 0 \Rightarrow \varphi = n\ \varphi_\delta$
Nonlinear optics	Positive refractive index	$\eta = \theta_o / \theta_i,\ \theta_o > \theta_i,$
	0 refractive index	$\eta = \theta_o / \theta_i,\ \theta_o = \theta_i,$
	Negative refractive index	$\eta = \theta_o / \theta_i,\ \theta_o < \theta_i,$

The second design library has several example metamaterials, references to their governing physical principles, expressions relating dimensions to properties or motions, and references to typical geometric implementations. Only two metamaterial classes are provided, for brevity. In the parametric models, **E** denotes elasticity tensor, ν is Poisson's ratio, κ is curvature, *l, d, t* represent length, diameter, and thickness dimensions, and *u* is displacement. The buckling surface images show a buckling disk in its two stable shapes with the curvature axes rotated 90 degrees from each other [13].

Table 2. Metamaterial design library

Type	**Sub-type**	**Physical principle**	**Parametric model**	**Image**
Auxetic	Linkage	Negative or 0 Poisson's ratio	$\mathbf{E} = \mathbf{E}(l,d,\text{topology}),$ $\nu = \nu(l,d,\text{topology})$	Figs 1b,2
	Chiral structure	Negative or 0 Poisson's ratio	$\mathbf{E} = \mathbf{E}(l,d,\text{topology}),$ $\nu = \nu(l,d,\text{topology})$	Fig 1c
Bistable mechanism	Buckling beam	Negative stiffness, Linear motion	$u = n\ \delta = f(l,t,d)$	Fig 3
	Buckling surface	Negative stiffness, Surface buckling	$\kappa = \kappa(D,t)$	

4 CONFIGURATION DESIGN METHOD

As stated in Section 1, the purpose of this paper is to help designers identify when metamaterials should be used during the design of an engineering system, rather than to design the metamaterial itself. The metamaterial is envisioned to be a component in a larger system being designed. The scope of the proposed configuration design method is the search for solutions to the functions in the function structure. The designer should choose a function, or a set of related functions, then start developing a solution. For that, s/he should identify physical and solution principles, develop behavior models, and select specific components and devices to implement the solution. Further, the method assumes that the designer may not be well acquainted with metamaterials so will likely select conventional physical principles and solution concepts. An insight that distinguishes this work as focused on metamaterials is as follows. Metamaterials exhibit behaviors that may seem impossible, or at least unusual. They present an apparent contradiction between what is needed (e.g., auxetic behavior) and what is familiar (e.g., positive Poisson's ratio). From that viewpoint, this work shares an interest in the contradictions that are central to Triz-based design methods [14]. The method then guides the designer to a behavior model, or a different physical principle, that meets the design requirements.

The proposed design method is listed in Figure 5. From a selected function, or set of functions, the designer identifies a candidate physical principle and develops a solution using that principle. A behavior model for the solution should be developed and quantified so that it can be evaluated analytically. That is, parameters and dimensions should be identified and related using analytical expressions based on the physical principle. The model should also integrate with the behavior model for the rest of the developing system solution, as appropriate. Then, the solution's behavior should be evaluated and compared against the design requirements.

If requirements are not met, then the design needs to be modified. Assuming the designer utilizes the previously selected physical principle (Step 9.a), then they should try to identify a parameter in the underlying math model of the principle such that a value for that parameter can be computed that enables the design requirement to be achieved. For example, an auxetic material has a negative Poisson's ratio; Poisson's ratio is the parameter associated with continuum mechanics (selected physical principle) and any negative value will enable lateral expansion of a material when it is under tension. A conventional approach involving continuum mechanics would have assumed a positive Poisson's ratio, which could not achieve simultaneous expansion under tensile loads. If the incorrect physical principle was identified originally, a new one should be investigated. The designer has already identified a solution principle and behavior model and quantified parameters in the previous design iteration. With this additional knowledge, they should be able to identify a more promising physical principle and adjust the behavior model accordingly. This would be case when considering a bistable mechanism and using Hooke's law governing solid materials as the governing physical principle. By changing to an intermittent motion principle, a bistable mechanism could be identified as a suitable solution and its behavior model developed by modifying the previous model. This method and the examples will be investigated in more detail in the next section.

For implementation, a library of physical principles should be developed, as well as a library of metamaterials, along with their behavior models. This work is underway but will not be reported here. For the purposes of this paper, only the metamaterials and physical principles discussed in Section 2 will be considered. Desired behaviors will be expressed using relationships similar to those in Section 2 and will be matched to the presented metamaterial behavior models to identify suitable metamaterials.

1. Identify candidate physical principle to implement the selected function(s)
2. Propose a solution concept based on the physical principle
3. Develop a behavior model
4. Identify and quantify physical parameters associated with relationships in the model
5. Evaluate behavior
6. Does device's behavior meet requirements?
    7. If so, then done
    8. If not, then
        9.a Identify relationships governing the physical principle's behavior OR 8.b Identify different physical principle that is closer to the desired behavior
    10. Identify parameter value of physical principle model that satisfies the requirement
    11. Find solution with that parameter value that achieves the desired behavior
    12. Update solution concept and return to Step 2

Figure 5: Configuration design method for metamaterials

5. EXAMPLES

5.1 Extending Beam

Assume that it is desired to design a beam that can expand by 20% in length but maintain a constant cross-section. Zero Poisson's ratio (0PR) materials could be used to maintain the constant cross section. The problem and proposed solution elements are shown in Figure 6. A scissors mechanism will be chosen to fulfill the extension requirement. However, as it extends, the mechanism contracts in the lateral (Y) direction. Any structures connected to the scissors will be deformed accordingly.

To maintain a constant cross-section, accommodations must be made for these lateral displacements. These requirements can be stated more precisely as: beam extension d_x causes lateral displacement $-d_y$. At cross-section boundaries, $u_y = u_z \cong 0$. To be able to match to the metamaterials, the requirement will be specified as a rule:

$d_x > 0 \Rightarrow u_y = -d_y \Rightarrow u_z = 0$

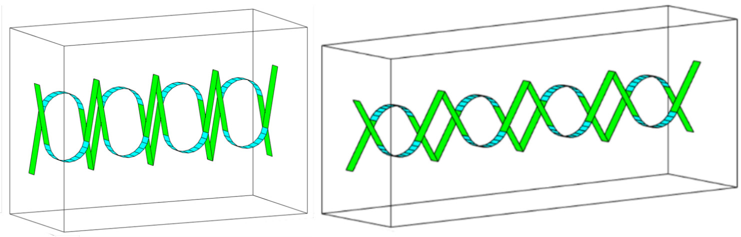


Figure 6. Extending beam example shown in beam's initial (left) and final (right) shapes

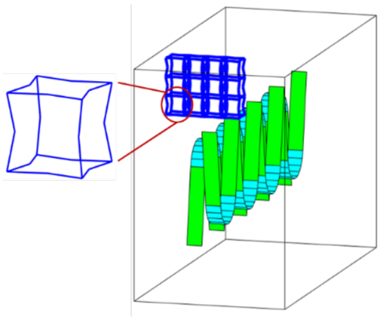


Figure 7. Extending beam example showing a partial row of 0PR unit cells

These requirements indicate that a 0PR material could be used to comprise the beam, since this requirement can be matched to the 0PR form of the auxetic behavior rule: u_y is not 0 and u_z is 0. That is, the scissors mechanism is the actuator that performs the extension, but the remaining volume within the beam is filled with 0PR lattice structure. Figure 7 shows an end-view of the beam with one row of 0PR lattice structures to illustrate the concept of connecting the scissors mechanism with the 0PR material to ensure the side and top walls maintain their positions.

5.2 Closure for Wearables

In the design of wearable devices, a requirement for good body fitting is often necessary. We are interested in developing a variety of assistive and rehabilitation devices for patients that augment or exercise joints, such as knees, elbows, and wrists. Such devices need to fit the patient's body parts well. Furthermore, for home usage, it must be easy for patients, who may have physical disabilities, to attach and remove the devices. Easy-to-use adjustments are needed.

We are developing a series of adjustable closures for these wearables. One type is meant to be adjustable by simply compressing the closure with a modest force to tighten it, then use body heat as the stimulus for a shape memory material to progressively release the closure. Stated more precisely, the requirement is that with a limited force, a series of discrete displacements is achieved. Mathematically, the tightening operation can be expressed as: $F_{th} > F_i > 0 \Rightarrow u = n\ \delta$. Since this is the same expression used to define the contraction/expansion with intermittent motion application of the bistable device, the progressive tightening operation can be performed by a series of bistable mechanisms connected in series. Note that the negative stiffness physical principle associated with bistable mechanisms is not needed for mechanism identification. A mock-up of a wearable on the calf with a 4-layer bistable mechanism to adjust tightness of fit is shown in Figure 8.

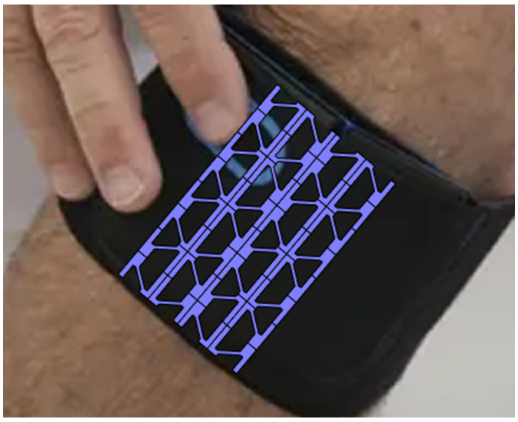


Figure 8. Bistable mechanism to adjust fit of wearable.

6. CONCLUSIONS

This paper introduced preliminary work towards a design method for identifying when metamaterials are suitable to use as a component in a developing system design. With their uncommon behaviors, metamaterials can be used to achieve functionalities that would otherwise be difficult to realize. Auxetic and bistable mechanisms were presented as example metamaterials and are related to their underlying physical principles. Snapshots of physical principle and metamaterial libraries were included to illustrate the type of information that should be available for designers to browse and use in the design method to configure solutions. Examples were presented to illustrate the usage of the design libraries.

Based on this work, simple qualitative physics models can be used to represent metamaterial behavior in a manner that facilitates identification of opportunities to utilize metamaterials in a developing design. If the behavior needed to achieve a function, or set of functions, is described using a similar qualitative model, physical principles and metamaterials can be identified. These qualitative models should represent forces, energy states, and resulting motions that describe essential metamaterial behaviors. Future work should explore additional metamaterials and governing physical principles to ensure broad applicability. More extensive design libraries should be developed, and a formal grammar developed to represent the qualitative physics models. The proposed configuration design method should be updated and tested to verify its correctness and usefulness.

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